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ELECTRIC MOTOR RELIABILITY MODEL

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Shaker Research Corporation

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ROME AIR DEVELOPMENT CENTER
Air Force Systems Command
Griffiss Air Force Base, New York 13441

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An existing data failure bank consisting of over 2000 actual fractional horsepower (FHP) motor failures was upgraded and utilized to develop a failure model for FHP motors. Motor failures were dominated by two failure modes; i.e., winding failures and bearing failures.

A mathematical reliability model was developed for each failure mode utilizing a Weibull cumulative distribution function and regression technique.

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Results of a literature survey of both failure modes were used to supplement failure data where required and verify the developed models.

Failure modes were recombined into a competing risk cumulative distribution model for use in predicting overall motor life and failure rates.

PREFACE

This final report was prepared by the Shaker Research Corporation of Ballston Lake, New York under Contract F 30602-76-C-0413. The work reported covers the period from September 1976 through August 1977. The report was prepared under direction of Mr. D. S. Wilson with major technical contributions from Mr. R. Smith of Shaker Research. Failure data collection was assisted by Mr. A. Fries of the Rotron Manufacturing Company. The RADC Project Engineer was Mr. L. Gubbins.

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EVALUATION

This contractual effort is part of the broad RADC Reliability Program intended to provide reliability prediction procedures for military electronic equipment and systems. These prediction procedures are contained in MIL-HDBK-217B for which RADC is the preparing activity. The final model developed in this study will replace the present electric motor reliability models that are now in MIL-HDBK-217B.

Lester J. Gubbins

LESTER J. GUBBINS
Project Engineer

1.0 INTRODUCTION

This report summarizes the results of a study directed towards the development of a failure rate prediction model for fractional horsepower electric motors. The objective of the study was to completely revise the information and failure rate models for fractional horsepower electric motors as used in electronic equipment currently included in Section 2.8.1 of MIL-HDBK-217B dated 20 September 1974.

To accomplish the program objectives, an existing failure data bank (Reference 1) of over 2000 failures of fractional horsepower motors, was updated and supplemented from a survey of electric motor manufacturers. The major portion of the failure data history was furnished by the Rotron Manufacturing Company. A regression analysis was conducted on all failures utilizing such variables as motor power, speed, temperature, failure mode, insulation type and motor type to establish the dominant motor life influencing parameters and their interrelationships.

Two competing failure modes were found to dominate motor life; i.e., bearing failures and winding failures. All bearings in the failure bank were of the grease-packed ball bearing type. Separation of failure by failure mode indicated that both failure modes could be described by a Weibull Commulative Distribution. A mathematical model for each failure mode with dominant variables was developed and a competing risk failure model established.

A literature survey of bearing failures and winding failures was conducted to verify failure modes and dependent failure variables. The following sections of this report describe the data failure bank, failure modes, literature review and development of the mathematical prediction models. Appendix A describes the regression analysis, and Appendix B outlines the recommended replacement for Section 2.8.1 of MIL-HDBK-217B.

2.0 SUMMARY

An existing data failure bank consisting of over 2000 actual fractional horsepower (FHP) motor failures was upgraded and utilized to develop a failure model for FHP motors. Motor failures were dominated by two failure modes; i.w., winding failures and bearing failures.

A mathematical reliability model was developed for each failure mode utilizing a Weibull cumulative distribution function and regression technique. Results of a literature survey of both failure modes were used to supplement failure data where required and verify the developed models.

Failure modes were recombined into a competing risk cumulative distribution model for use in predicting overall motor life and failure rates.

3.0 DISCUSSION

The development of a "Reliability Model" for fractional horsepower (FHP) motors necessitates a well-documented data bank of motor failures on which to base failure predictions. To develop such a data bank, 47 manufacturers of fractional horsepower motors were contacted for assistance. In addition, a literature search was conducted through the Engineering Index (EI) and the National Technical Index Service (NTIS) covering motors, windings, insulation, bearings and failure modes. As a result of these efforts over 3000 failure histories were collected with 1614 failures suitably documented for analysis purposes. The major portion of the failure bank was furnished by the Rotron Manufacturing Company.

The composition of motor types encompassed by the failure bank covers power ranges from 15 watts input to 625 watts input. Motor types encompassed:

Synchronous polyphase motors	Shaded pole motors
Polyphase induction motors	Split phase motors
Capacitor-run motors	Capacitor-start motors

All motors analyzed contained grease packed, single and double shielded ball bearings to Antifriction Bearing Engineers Committee (ABEC) quality 3 and 5. Motor (NEMA) insulation classes F and H corresponding to IEEE classes 150°C and 180°C are the only classes used in the failure bank.

Operating parameters of the motors cover the steady state operating range from 18°C to 140°C and thermal cycling from -65°C to 125°C. Operating speeds varied from 1750 RPM to 23000 RPM covering two input frequencies; i.e., 60 Hz and 400 Hz. Motor input voltage included 115V, 208V, 230V, 416V and 440V.

Because of the large number of variables, all data was stored in a digital computer with the following information available for each test point:

- Failure mode
- Time to failure
- Date of failure
- Motor type
- Bearing class
- Bearing lubricant
- Insulation class
- Ambient temperature
- Bearing temperature rise
- Motor temperature rise
- Motor load
- Motor frame size
- Special construction feature

3.1 Data Bank Analysis

The major portion of the failure histories had previously been analyzed (Reference 1) for failure modes with the following results.

Bearing failures	80.85%
Electrical failures	16.55%
Mechanical failures	2.60%

These results clearly indicated that a failure model for electric motors could be developed utilizing two basic failure modes; i.e., bearing failures and winding failures. In a normal population of motors, it would be anticipated that the dominant failure mode would be the result of bearing failures. This condition, however, is limited to non-regreaseable ball bearing motors. As a result of these considerations only bearing and electrical failures were analyzed from the data bank.

Failure data are dominantly composed of life test results of individual test populations. It was found (1) that the most effective way of handling the data was by means of a Weibull Cumulative Distribution analysis of each individual test population. The results of this analysis provides a linear regression best fit Weibull slope (β) and characteristic life (α) for each test group of motors. Additional regression techniques are applied to determine the influence of parameters such as temperature, speed, bearing lubricant, motor type, etc. on characteristic life.

Since a failure model of bearing failures as well as winding failures is desirable, it is necessary to separate the two failure modes. Direct separation from test groups influences the order ranking of failures inducing errors in the predictions of Weibull slope and characteristic life. A technique for separating failure modes as proposed by Nelson (2) is to consider all failures of one failure mode as suspended (censored) test units when analyzing a second failure mode. This is accomplished by means of a Weibull Cumulative Hazard function. Censoring of failures of all other failure modes except the failure mode of interest was applied to each test group population during the data analysis. The reader is referred to Reference 2 for a comprehensive discussion of censoring and only the definition of the cumulative hazard function is provided; i.e.

Weibull Cumulative Distribution Function is defined as:

$$F(t) = 1 - e^{-(\frac{t}{\alpha})^{\beta}} \qquad (1)$$

Probability Density Function is defined as:

$$f(t) = \frac{d}{dt}(F(t)) = \frac{\beta}{\alpha\beta}\, t^{\beta-1} e^{-(\frac{t}{\alpha})^{\beta}} \qquad (2)$$

Hazard Function (failure rate) is defined as:

$$h(t) = \frac{f(t)}{1-F(t)} = \frac{\beta}{\alpha\beta}\, t^{\beta-1} \qquad (3)$$

Cumulative Hazard Function is defined as:

$$H(t) = \int_{0}^{t} h(t)dt = (\frac{t}{\alpha})^{\beta} \qquad (4)$$

A further data separation was found desirable in tabulation of the results of the analysis; i.e., tabulation of bearing failures by grease type. Three basic greases were utilized for the majority of the tests. Approximately 30 additonal grease types were tested in motors, but the majority of the greases did not have sufficient test points to provide statistically significant results.

Tables 1 through 4 summarize the Weibull Characteristic life and slope for three different greases and for windings. These tables list some of the significant test parameters such as temperature and speed. An additional parameter that requires further definition is indicated in the tables as quality code. The 0-Code refers to commercial standards for motor construction while the 1-Code is of military quality. Although considerably more information is available for each data point, only the more significant parameters as determined from analysis discussed in Section 3.2.2 are tabulated.

In order to test the data fit to a Weibull distrubition, the correlation coefficient and an F test was computed for each test population group. Table 5 is a typical output from 45 test populations. Since a correlation coefficient of 1 is perfect correlation of data fit to a straight line, values of 0.9 and greater indicate a high degree of correlation of the data utilizing the Weibull Distribution Function.

The correlation coefficient is not sufficient for testing the "goodness of fit" when the sample size is small. As the number of test points is reduced, this measure of fit is biased toward an unusually high value. In fact, (R) is exactly 1 when there are two data points being fit. An unbiased measure of the fit is provided by the F-test when the sample size is small. The F - test, which accounts for the number of data points being fit, was used in measuring the "goodness of fit" when the number of test points was small. By inspection the F-test (t-test for this single parameter fit) shown in Table 5 reveals the Weibull function is a highly reliable explainer of the data contained in forty-five tests shown. Note that the F-value is high (typically 10 or more) for the majority of tests. The significance level of the Weibull fit exceeds the 95th percentile for most of the data displayed.

The motor failure prediction models finally developed are based on the summarized data of Tables 1 through 4. This data, therefore, was stored in a digital computer and subjected to the analysis described in Appendix A and summarized in Section 3.2.2 of this report.

TABLE 1
LIFE TEST SUMMARY
GREASE 1

TEST AMB	TEMP BRG	BORE DIA	SPEED HNDRS	NUMBER OF TEST UNITS		QUAL CODE	WEIBULL SLOPE	CHARACTER. LIFE
C	C	MM	RPM	TOTL	#FAIL	#	BETA	HOURS
140	182	3.18	205	10	5	1	2.97	204
125	169	3.18	110	30	13	1	1.72	8261
125	177	3.18	205	25	18	1	1.71	8735
125	177	3.18	205	4	3	0	1.89	3656
125	177	3.18	205	20	14	0	3.27	3473
125	167	3.18	205	20	6	0	6.14	3927
125	163	4.76	225	3	3	1	0.85	4439
125	163	4.76	225	4	4	1	2.20	5001
125	163	4.76	225	5	4	1	5.71	3661
125	163	4.76	225	4	4	1	4.06	6419
125	190	4.76	210	3	3	1	2.52	2340
125	163	4.76	225	5	5	1	2.18	6057
125	163	4.76	225	10	10	1	1.46	3562
125	163	4.76	225	5	5	1	2.26	9033
100	138	4.76	225	5	4	1	1.30	10111
125	167	3.18	205	17	12	1	4.92	6312
125	167	3.18	205	5	5	1	1.67	6800
50	87	3.18	200	7	3	1	7.38	61380
125	167	3.18	205	12	11	1	4.99	9935
24	41	6.35	35	3	3	0	1.11	30811
125	160	6.35	60	5	5	0	13.43	4501
125	193	6.35	117	5	4	0	2.56	2273
125	160	6.35	60	5	5	0	3.32	4533
125	160	6.35	60	5	5	0	2.65	5610
125	160	6.35	60	4	4	0	4.62	6558
125	160	6.35	60	3	3	0	10.51	6044
125	145	6.35	110	9	9	0	2.11	4633
125	145	6.35	110	5	5	0	2.96	4786
125	165	6.35	55	10	9	1	5.65	3687
72	112	6.35	220	5	3	1	1.63	13533
100	110	6.35	103	6	6	1	5.33	27191
125	165	6.35	55	6	6	1	2.34	4577
50	65	6.35	33	15	8	1	1.96	95907
100	120	6.35	35	14	12	1	1.45	40842
100	135	3.18	32	30	20	1	3.07	43975
100	135	3.18	32	12	7	1	2.95	38352
72	109	4.76	225	6	6	1	2.94	42862
100	137	4.76	225	8	7	1	4.85	11646
125	162	4.76	225	10	10	1	2.90	6273
72	105	4.76	31	23	23	1	2.05	24848

TABLE 1 CON'T

TEST AMB	TEMP BRG	BORE DIA	SPEED HNDRS	NUMBER OF TEST UNITS		QUAL CODE	WEIBULL SLOPE	CHARACTER. LIFE
C	C	MM	RPM	TOTL	#FAIL	#	BETA	HOURS
72	105	4.76	31	12	11	1	2.25	25136
72	105	4.76	31	6	5	1	4.54	29433
72	105	4.76	31	8	8	1	0.80	32359
125	151	3.18	225	4	4	1	1.39	4544
100	135	3.18	32	12	10	1	2.18	41217
125	162	3.18	200	6	4	1	0.87	7342
50	88	4.76	200	24	4	1	0.70	765796
72	117	3.18	205	4	4	1	8.56	14307
100	135	3.18	32	12	8	1	1.65	47431
100	135	3.18	32	15	15	1	1.94	29864
72	107	3.18	32	12	7	1	2.08	39831

TABLE 2
LIFE TEST SUMMARY
GREASE 2

TEST AMB	TEMP BRG	BORE DIA	SPEED HNDRS	NUMBER OF TEST UNITS		QUAL CODE	WEIBULL SLOPE	CHARACTER. LIFE
C	C	MM	RPM	TOTL	#FAIL	#	BETA	HOURS
125	167	3.18	205	5	5	0	4.81	3341
125	167	3.18	205	5	5	0	8.14	4036
72	110	4.76	33	5	5	1	1.72	41850
72	92	6.35	35	7	7	0	0.98	17560
72	114	6.35	112	3	3	0	1.62	2752
72	89	6.35	33	3	3	1	6.00	26750
72	89	6.35	33	3	3	1	3.84	8133
72	92	8.10	33	5	5	1	5.34	11994
72	105	4.76	31	7	6	1	4.93	29966
72	105	4.76	31	7	7	1	4.58	36531
72	105	4.76	31	7	7	1	4.55	37944
72	100	3.18	105	4	4	1	1.37	22081
72	110	4.76	33	6	3	0	1.28	16938
72	114	3.18	33	5	4	1	2.18	17344
72	110	4.76	225	4	4	1	2.87	21322
72	110	4.76	33	6	3	0	1.28	16938
72	92	8.10	33	6	6	0	2.45	3936

TABLE 3
LIFE TEST SUMMARY
GREASE 3

TEST AMB C	TEMP BRG C	BORE DIA MM	SPEED HNDRS RPM	NUMBER OF TEST UNITS TOTL	#FAIL	QUAL CODE #	WEIBULL SLOPE BETA	CHARACTER. LIFE HOURS
125	167	3.18	205	14	9	1	3.63	9352
125	167	3.18	205	32	30	0	2.41	4054
125	177	3.18	205	37	28	0	3.41	2854
125	167	3.18	205	33	26	0	2.61	3109
125	177	3.18	205	13	8	0	3.91	2297
125	177	3.18	205	5	3	0	1.57	784
125	167	3.18	205	17	8	0	2.81	2799
125	167	3.18	205	10	4	0	3.62	4338
125	177	3.18	205	5	4	1	35.61	1858
125	177	3.18	205	7	5	1	4.34	2694
125	177	3.18	205	6	4	1	2.49	1687
125	160	4.76	135	3	3	0	3.85	1440
125	163	4.76	225	5	5	0	1.26	1588
125	163	4.76	225	5	5	0	4.07	1066
125	163	4.76	225	8	7	0	1.24	537
125	163	4.76	225	3	3	0	1.17	872
125	158	4.76	110	6	6	0	1.79	3740
125	163	4.76	225	4	4	0	3.60	1081
125	163	4.76	225	5	5	1	3.51	2647
125	163	4.76	225	4	4	0	0.76	861
125	163	4.76	225	5	5	0	1.26	1588
125	163	4.76	225	5	5	0	4.07	1066
125	163	4.76	225	7	6	0	0.93	918
72	152	4.76	165	4	4	1	0.84	3237
125	163	4.76	225	4	4	0	5.80	676
110	148	4.76	33	3	3	0	9.02	8669
125	150	6.35	110	4	4	0	1.46	380
125	160	6.35	60	4	4	0	1.97	4345
125	214	6.35	100	5	5	0	2.26	618
135	170	6.35	55	5	5	0	3.21	1130
135	170	6.35	55	5	5	0	4.19	1530
125	214	6.35	100	5	5	0	2.90	591
135	201	6.35	55	5	5	0	3.24	1611
125	214	6.35	100	3	3	0	2.37	1170
135	203	6.35	60	5	5	0	3.91	1868
125	155	6.35	35	6	6	0	3.01	4006
125	145	6.35	75	6	6	0	3.32	2681
125	160	6.35	60	7	7	0	4.54	853
125	193	6.35	118	3	3	0	4.48	186
125	193	6.35	118	3	3	0	1.23	1371

TABLE 3 CON'T

TEST AMB	TEMP BRG	BORE DIA	SPEED HNDRS	NUMBER OF TEST UNITS		QUAL CODE	WEIBULL SLOPE	CHARACTER. LIFE
C	C	MM	RPM	TOTL	#FAIL	#	BETA	HOURS
72	140	6.35	100	7	7	0	3.01	301
125	160	6.35	50	6	5	0	1.99	146
125	214	6.35	100	3	3	0	2.73	584
125	214	6.35	100	5	5	0	1.69	396
125	160	6.35	60	4	4	0	2.91	1585
130	165	6.35	60	15	14	0	4.39	295
125	160	6.35	60	3	3	0	1.14	6663
125	160	6.35	60	5	5	0	1.99	5456
125	160	6.35	60	5	5	0	2.04	4155
125	160	6.35	60	15	15	1	2.98	1417
125	175	3.18	205	3	3	0	1.08	1431
125	200	3.18	210	3	3	0	1.01	879
125	200	3.18	210	3	3	0	1.57	831
125	175	3.18	205	5	5	0	3.68	1746
125	192	3.18	205	3	3	0	0.73	1444
72	105	4.76	31	15	14	1	3.97	18574
72	105	4.76	31	15	14	1	2.55	24438
72	105	4.76	110	5	5	1	1.25	10604
140	182	3.18	205	16	4	1	1.84	855
125	177	3.18	205	7	7	0	1.59	4023
125	177	3.18	225	4	3	0	1.56	4925
125	163	4.76	225	33	25	0	1.55	1883
125	175	3.18	205	8	6	0	1.32	2474
125	163	4.76	225	12	7	0	1.96	1038
125	150	4.76	225	7	7	0	1.49	3286
125	163	4.76	225	4	4	0	3.21	862
125	163	4.76	225	6	5	1	1.35	2127
125	175	3.18	200	5	3	0	1.65	3312

TABLE 4
LIFE TEST SUMMARY
WINDING FAILURES

TEST AMB	TEMP BRG	BORE DIA	SPEED HNDRS	NUMBER OF TEST UNITS		QUAL CODE	WEIBULL SLOPE	CHARACTER. LIFE
C	C	MM	RPM	TOTL	#FAIL	#	BETA	HOURS
125	177	3.18	205	4	4	1	1.47	452
140	182	3.18	205	17	16	1	0.98	1348
140	182	3.18	205	12	11	1	0.74	726
125	177	3.18	205	7	6	1	2.65	4569
125	177	3.18	205	6	5	1	1.36	4359
140	192	3.18	205	6	6	1	2.30	2689
140	182	3.18	205	10	5	1	2.41	2516
125	169	3.18	110	30	17	1	1.35	8124
125	177	3.18	205	25	7	1	1.62	15373
125	167	3.18	205	29	29	1	2.51	14527
125	167	3.18	205	14	5	1	4.09	9568
125	167	3.18	205	33	3	0	4.45	2306
125	167	3.18	205	5	4	0	1.09	182
125	167	3.18	205	13	3	0	0.77	4702
125	167	3.18	205	17	7	0	0.75	2742
125	167	3.18	205	10	6	0	1.02	4073
125	177	3.18	205	20	6	0	3.63	4720
125	167	3.18	205	20	10	0	3.77	4531
125	167	3.18	205	5	3	0	9.29	6545
125	167	3.18	205	6	5	1	1.91	7429
140	174	3.18	110	12	10	1	3.13	411
140	174	3.18	110	14	14	1	3.97	408
125	167	3.18	205	17	5	1	4.11	7930
125	177	3.18	205	30	16	1	1.82	11270
50	87	3.18	200	8	3	1	0.49	230883
24	66	3.18	205	15	4	1	0.51	716959
50	92	3.18	205	15	4	1	1.04	60025
100	135	3.18	32	30	10	1	1.14	20023
140	185	3.18	115	5	5	1	18.22	113
125	170	3.18	115	10	9	1	3.22	1890
125	170	3.18	115	12	12	1	1.07	168
152	180	3.18	145	6	6	1	3.26	1532
140	168	3.18	145	11	10	1	3.70	8397
140	182	3.18	205	16	9	1	1.91	1476
125	163	4.76	225	33	4	0	0.68	15334
50	87	3.18	200	24	4	1	0.50	999999
125	163	4.76	225	5	5	0	4.13	564

TABLE 5

NUMBER OF TEST UNITS		QUAL CODE	WEIBULL SLOPE	CHAR. LIFE	CORR COEF	F TEST VALUE
TOTL	#FAIL	##	BETA	HOURS	##	##
14	9	1	3.63	9352	.983	204.1
32	30	0	2.41	4054	.954	286.3
37	28	0	3.41	2854	.970	423.0
33	26	0	2.61	3109	.903	106.7
13	8	0	3.91	2297	.972	102.9
5	3	0	1.57	784	.956	10.7
17	8	0	2.81	2799	.949	55.2
10	4	0	3.62	4338	.989	91.6
5	4	1	35.61	1858	.901	8.6
7	5	1	4.34	2694	.949	27.6
6	4	1	2.49	1687	.884	7.2
3	3	0	3.85	1440	.908	4.7
5	5	0	1.26	1588	.940	23.0
5	5	0	4.07	1066	.908	14.1
8	7	0	1.24	537	.976	103.5
3	3	0	1.17	872	.995	118.1
6	6	0	1.79	3740	.669	3.2
4	4	0	3.60	1081	.937	14.6
5	5	1	3.51	2647	.965	40.9
4	4	0	0.76	861	.867	6.0
5	5	0	1.26	1588	.940	23.0
5	5	0	4.07	1066	.908	14.1
7	6	0	0.93	918	.813	7.8
4	4	1	0.84	3237	.975	39.5
4	4	0	5.80	676	.891	7.7
3	3	0	9.02	8669	.999	532.1
4	4	0	1.46	380	.825	4.2
4	4	0	1.97	4345	.997	365.3
5	5	0	2.26	618	.982	84.8
5	5	0	3.21	1130	.802	5.4
5	5	0	4.19	1530	.904	13.4
5	5	0	2.90	591	.882	10.5
5	5	0	3.24	1611	.966	42.5
3	3	0	2.37	1170	.953	9.9
5	5	0	3.91	1868	.968	45.6
6	6	0	3.01	4006	.957	43.7
6	6	0	3.32	2681	.919	22.0
7	7	0	4.54	853	.955	52.2
3	3	0	4.48	186	.987	38.4
3	3	0	1.23	1371	.927	6.1
7	7	0	3.01	301	.991	292.3
6	5	0	1.99	146	.949	27.6
3	3	0	2.73	584	.970	16.4
5	5	0	1.69	396	.949	27.3
4	4	0	2.91	1585	.945	17.0

3.2 Bearing Reliability Model

As previously discussed, the reliability model to be developed for FHP motors is dominated by two failure modes -- bearings and windings. Weibull characteristic life and slope at different operating temperatures and conditions were calculated for each failure mode. The approach followed was to develop a mathematical model of the influence on life of parameters such as temperature, speed, motor type, etc. for each failure mode. An expression could then be developed to re-combine both failure modes into a single motor failure model. The following section discusses the development of a failure model for the motor bearing, commencing with a literature review, developing the model parameters of concern and summarizing the bearing data analysis conducted.

3.2.1 Literature Review of Bearings

In order to assist in the data analysis, a literature review of grease lubricated bearings was conducted. The purpose of such a review was to insure that all of the significant bearing parameters that might influence bearing life were encompassed in the bearing model. Furthermore, since all motor failure data was run under light bearing load conditions, it was desirable to obtain trends from literature of load influences on bearing life. Literature was also reviewed to obtain experience in the operation of grease bearings at low temperature since data bank data in that range is somewhat limited.

3.2.1.1 Load Influences on Bearing Life

It is commonly accepted that a lubricant film must be developed between rolling elements of a bearing in order to insure long-life operation. This film is termed an elastohydrodynamic lubrication (EHL) film for rolling element bearings.

A generalized elastohydrodynamic theory for film clearance in rolling contact bearings is attributed to Dowson (22). He expresses film thickness H as:

$$H = \frac{1.6\ G^{0.6}\ U^{0.7}}{W^{0.13}} \quad (5)$$

Each of the terms in Equation (5) are described by additional equations which are not described in this discussion since the interest is in load W. A number of sources of the EHL film prediction analysis are available. This expression, however, was selected since it clearly illustrates the influence of load W on film thickness. The reciprocal of load is raised to the 0.13 power compared to the other terms in the equation. Since lubricated related surface distress is associated with operation in the absence of an EHL film, ball bearing life in terms of surface distress is a function of film thickness. Loss of grease film in a bearing due to load would lead directly to reduced life. Equation (5) suggests that bearing load will not significantly influence film thickness until loads approaching the maximum capability of the bearing are approached.

Experiments conducted by Poon (3) indicate that grease film thicknesses after run-in are generally less than oil films. These experiments do verify however, the existence of a grease film in the contact region. The development of such a film is undoubtedly a hydrodynamic type of action. Therefore, it is anticipated that the influence of load on film for grease lubrication is similar to that given in Equation (5) where speed contained in the (U) term is an important parameter since it is raised to the 0.7 power. In practice, Scarlett (4) indicates few grease lubricated bearings operate at loadings that are critical and load can generally be ignored. It is stated however, at higher speeds grease does not act as a good coolant and load influences become important. Therefore, heat generated at higher speeds is important in reducing lubricant viscosity (μ_o). New Departure (5) provides a nomograph that relates the life reduction due to load on grease lubricated bearings. The nomograph was reduced to equation form for deep groove bearings utilized in fractional horsepower motors where the inner race rotates, as follows:

$$\text{Life multiplier } M = 1 - \left(\frac{N}{933}\right)^{.69} \frac{W}{SP} \qquad (6)$$

where:

N = RPM

W = Applied load (lbs)

SP = Specific dynamic capacity at 33-1/3 RPM (lbs)

To obtain an insight into the influence of load from this equation, consider a typical FHP motor bearing, i.e.: 9 mm bore. The specific dynamic catalog capacity of this bearing is, SP = 485 lbs. Except for gear or belt drive motor applications, the typical bearing load is in the range of 2 pounds. From Equation (6):

$$M = 1 - \left(\frac{N}{933}\right)^{.69} \frac{2}{485}$$

The normal speed range for a FHP motor is 900 RPM to 20,000 RPM maximum. The life multiplier ranges from M = .996 to .966.

The correction to life over the operating range is less than four percent and could be ignored. If however the bearing is loaded to 50 percent of specific load, life is reduced 51 percent at 900 rpm and approaches zero at 2500 rpm. Therefore, high loads introduce significant reductions in bearing life.

In order to check the accuracy of this load factor, a comparison was made with the following data taken from Reference 6:

Bearing 204 (20 mm bore)

Speed 10,000 RPM

Amb 232°C

Axial Load	Life	Predicted Life
40 lbs.	1000 hrs.	1000 hrs.
230 lbs.	100 hrs.	293 hrs.
330 lbs.	10 hrs.	10 hrs.

The prediction of life assumed 1000 hours life at 40 lbs. load and predicted the reduction in life at 230 and 330 lbs. The correlation of life reduction with load checks quite well with Reference (6). Other direct correlations of load influences on life at constant temperature and speed were not found and therefore Equation (6) was selected as a suitable expression to relate load influences on life.

Reference (1) developed a relationship of the influence of temperature of the bearing on life in the form:

$$\text{Log life} = \frac{A}{T} + B \qquad (7)$$

where: A and B are constants

T = Bearing absolute temperature

It is desirable to describe the influence of load on life in logarithmic form to provide compatibility with Equation (7). A close approximation of Equation (6) in log form is:

$$- .001N \left[\frac{W}{SP}\right]^{1.5} = \text{Log } M \qquad (8)$$

The generalized life formula of Equation (7) would thus be modified by Equation (8) i.e.:

$$\text{Log life} = \frac{A}{T} + B - .001N \left[\frac{W}{SP}\right]^{1.5} \qquad (9)$$

3.2.1.2 Low Temperature Influences on Bearing Life

As the operating ambient temperature of a bearing is decreased, eventually a temperature must be attained where the lubricant can no longer perform its function because it approaches a solid state. It would be anticipated that the lubrication mechanism does not cease at an exact temperature but is subject to a transition range where lubrication becomes marginal. Following this assumption, a temperature must exist where the life of the bearing no longer increases but begins to decrease with decreasing temperature. The literature review provided no available data of grease lubricated bearing life at low temperature. Low temperature testing by Crisp and Wells (7) resulted in occasional damage in the bearing after short periods of operation at ambients in the range of -38°C. They concluded that damage noted in the sliding regions of the bearing was indicative of marginal lubrication; however, there was no indication of the anticipated life. Other investigators (8,9) have been concerned with bearing torque at low temperatures. In all torque measurement tests of greases at low temperature, a temperature is reached where torque increases rapidly with decreasing temperature. If Crisp (7) is correct that the increasing torque level may be associated in part with marginal lubrication. then the transition temperature to high torque operation may be indicative of reduced lubrication and bearing life. At very low bearing speeds increasing torque appears to be a function of lubricant properties (9) and lubricant shear resistance. As speeds are increased, the onset temperature of increasing torque increases (7) and at DN values of 7000 a transition temperature of 16°C outer race temperature was measured. If this is due in part to marginal lubrication, then one might expect bearing life of FHP motors to begin to fall off at bearing temperatures in the range of 16°C. Since literature concerned with life at low temperature of grease packed bearings is so limited, considerable conjecture is associated with estimating the lower temperature bound of Equation (7). Although actual life test data from the data bank is used to establish the useful range of Equation (7), it is desirable to verify these trends from available literature. The trends worthy of note from this review are that reduction in bearing life with temperature may occur at rather high ambient temperatures, and the transition point is a function of bearing speed and lubricant characteristics. The reader is referred to Section 3.2.2.4 for a discussion of experimental results regarding low temperature life of bearings.

3.2.2 Development of Bearing Life Prediction Model

The bearing life prediction model was developed from the data tabulated in Tables 1 through 3 utilizing a multivariable regression analysis computer program. This program and its capabilities is described in Appendix A.

Reference 1 established bearing grease life as strongly influenced by temperature in the form of Equation (7):

$$\text{Log life} = \frac{A}{T} - B \qquad (7)$$

This equation linearizes the relationship between life and temperature permitting the use of linear regression techniques to establish the coefficients A and B. To illustrate the relationship of Equation (7), Figure 1 is a plot of the log of characteristic life of all data from the data bank including the data of Tables 1 through 3 as a function of the reciprocal of temperature. Without introducing any additional variables, the strong dependency of life on temperature is apparent. A linear regression of this data results in a solution of Equation (7) as :

$$\text{Log life}_B = \frac{2342}{T} - 1.908 \qquad (10)$$

where: Life = Weibull characteristic life (α_B) in hours
T = Absolute temperature ^oK

This equation explains 54 percent of the data scatter of Figure 1. A further reduction in scatter can be effected by including additional variables that influence life. This implies that the coefficient B of Equation (7) is composed of a number of additional variables.

The variables entered into regression included:

Bearing operating temperature	Quality code and grease type
Bearing speed	Winding temperature rise
Bearing size	Motor voltage
Number of motor poles	Motor winding size

The standard F-test was used to determine the significance of correlation between the observed test failure lives and the various operating parameters listed. The dependent variables on bearing life were:

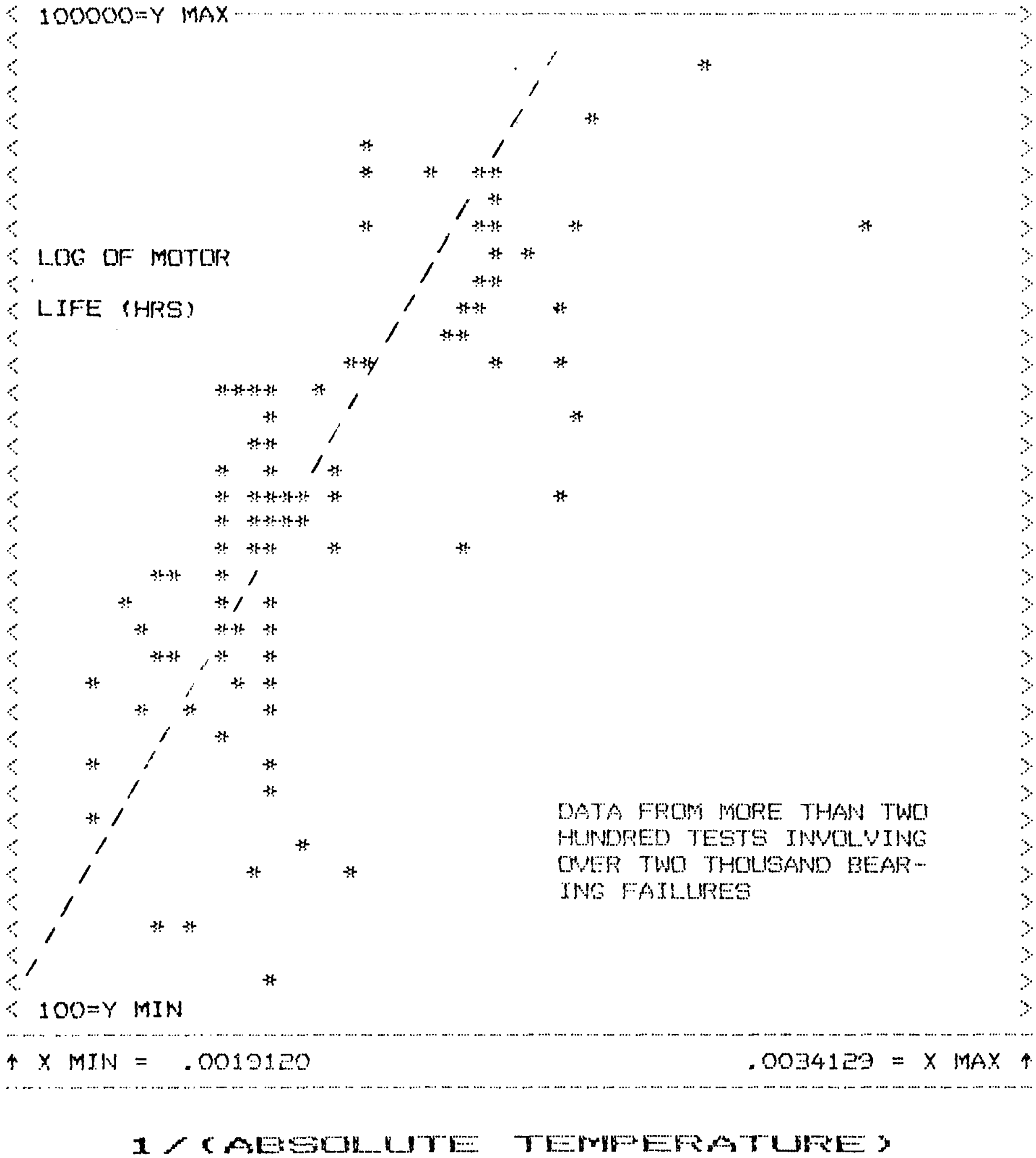


Figure 1

FRACTIONAL HORSEPOWER MOTOR
RELIABILITY DATA
LIFE VERSUS TEMPERATURE

Bearing temperature

Bearing size (bore diameter in mm) times speed

Quality code

Grease type

In separating grease type in the regression analysis, it was observed that the slope coefficient A of Equation (7) varied for each grease ranging from 1730 to 2441. Although one might anticipate that different greases might have a different slope of life versus temperature, the distribution of failures over the temperature range for two greases was skewed. This tended to reduce the reliability of the slope prediction for individual grease. As a result, the slope for all greases was maintained at 2342 from Equation (10) and the influence of quality code, speed, and grease type was evaluated.

Using Equation (4) for illustrative purposes, the equation is rewritten in the following form to obtain the influence of quality code:

$$\text{Log life}_B - \frac{2342}{T_B} = \begin{pmatrix} x_1 \text{ qual code: } 0 \\ x_2 \text{ qual code: } 1 \end{pmatrix} \qquad (11)$$

The mean and standard deviation of x_1 and x_2 values are calculated. These corrections are then transposed to the left-hand side and the influence of speed determined in the same manner; i.e.

$$\text{Log life}_B - \frac{2342}{T_B} - \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_3 \text{ DN} + x_4 \qquad (12)$$

A regression is run on this equation to determine the coefficients x_3 and x_4. These coefficients are then transposed to the left side of the equation and the influence of each grease obtained:

$$\text{Log life}_B - \frac{2342}{T_B} - \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - x_3 \text{ DN} - x_4 = \begin{pmatrix} x_5 \\ x_6 \\ x_7 \end{pmatrix} \qquad (13)$$

As the coefficients are determined, the predictive equations can be rewritten:

$$\text{Log life}_B = \frac{2342}{T_B} - 1.91 \qquad (10)$$

$$\text{Log life}_B = \frac{2342}{T_B} - 1.91 + q \qquad q = \text{quality code} \qquad (14)$$

$$\text{Log life}_B = \frac{2342}{T_B} - 1.91 + q - eDN \qquad e = \text{constant} \qquad (15)$$

$$\text{Log life}_B = \frac{2342}{T_B} - eDN + q - k_g \qquad K_g = \text{grease constant} \qquad (16)$$

The data was tested against each of these equations and percentage of actual lives falling within one standard deviation were calculated. A standard deviation in log form of .35 for the intercept coefficient B of Equation (7) was used. Table 6 summarizes the equations and data fit.

The general equation for bearing characteristic life (hrs) may be written in the form:

$$\text{Log life}_B = \frac{2342}{T_B} + q - 4.32\ DN \times 10^{-6} - K_g \qquad (17)$$

where: q = quality code

commercial quality $q = -.27$

Military spec. quality $q = +.12$

DN = Bearing bore (mm) x speed (rpm)

K_g = Grease constant per Table 7

e = 4.32×10^{-6} a constant

TABLE 7

Grease Constant K_g

Grease	K_g
1	1.35
2	1.55
3	1.74

TABLE 6

Test of Bearing Failure Data

Versus

Predictive Equations for One Standard Deviation**

	Equation	Percent of Data Within One Standard Deviation
A.	$\frac{2342}{T_B} - 1.91$	69.85
B.	$\frac{2342}{T_B} - \begin{pmatrix} 2.03\ LQG^* \\ 1.64\ UQG \end{pmatrix}$	70.59
C.	$\frac{2342}{T_B} - \begin{pmatrix} 1.77\ LQG \\ 1.38\ UQG \end{pmatrix} - 4.32 \times 10^{-6} DN$	74.26
C	$\frac{2342}{T_B} - \begin{pmatrix} -\ .27\ LQG \\ +\ .13\ UQG \end{pmatrix} - 4.32 \times 10^{-6} DN - \begin{pmatrix} 1.50\ GR1 \\ 1.70\ GR2 \\ 1.89\ GR3 \end{pmatrix}$	75.00

* LQG = low quality grade
UQG = upper quality grade

** One standard deviation was found to be .35 on the log life (hrs) scale

The data bank did not contain sufficient bearing load variation to include load influences on life predictions. It was necessary to develop a load-life modifier from available information found in literature. As discussed in Section 3.2.1.1, Equations (4) and (5), Equation (17) may be rewritten to include load effects.

$$\text{Log life}_B = \frac{2342}{T_B} + q - 4.23DN \times 10^{-6} - Kg - .001N \left[\frac{W}{SP}\right]^{1.5} \qquad (18)$$

The useful temperature range of this equation varies from the maximum recommended temperature by the grease manufacturer to a temperature in the range of 15°C. Section 3.2.2.4 should be referred to for development of the low temperature life characteristics.

3.2.2.1 Bearing Temperature Rise

Equation (18) was developed for predicting life requiring considerable knowledge regarding the bearing; i.e. the type of grease, the bearing temperature, bearing size and speed. Since bearing temperature is the dominant life influence, accurate determination of this temperature is quite important.

Motor bearing temperature is a difficult parameter to ascertain. Winding temperature rise above ambient temperature is often a good indicator of bearing temperature. Generally, bearing temperatures (outer race) run 10°C cooler than windings. In some type of motors however, the bearings may run hotter than the winding depending on their cooling and location in the motor. Table 8 illustrates some typical measurements made on a number of different type motors. As noted, bearing temperature rise above ambient ranges from 15° to 150°C. The distribution of motor winding temperature rise taken by resistance measurements for a hundred and seventy-five different fan-cooled FHP motors, ranging from polyphase to capacitor-run and shaded poles is presented in Figure 2. The mean value for winding rise is 36.4°C. The mean value for bearing temperature rise would be 26.4°C. The standard deviation is approximately 16°C. Therefore, one can anticipate that a 68 percent probability exists that the bearing temperature will fall in the range

```
 STEP              NORMALIZED DATA HISTOGRAM

 4.000 X
 8.000 X
12.000 X
16.000 XXXXXXXXXXXX
20.000 XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
24.000 XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
28.000 XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
32.000 XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
36.000 XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
40.000 XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
44.000 XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
48.000 XXXXXXXXXXXXXXXXXXXXXXXXX
52.000 XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
56.000 XXXXXXXXXXXXXXX
60.000 XXXXXXX
64.000 XXXXXXX
68.000 XXXXX
72.000 XXXXXXXXXX
76.000 XXXXXXX
80.000 XX
84.000 XX
88.000 XX
92.000 X
96.000 XX
00.000 AREA = 175.0        STD DEV = 015.8507 PEAK =  020.
```

Figure 2

Motor Temperature Rise Distribution

of 10° to 43°C. Recognizing that there is approximately a 50 percent reduction in life for every 10°C rise in temperature, this range is quite broad. The conservative approach when the temperature is not known is to use a bearing temperature rise of 40°C. This temperature agrees with the data failure bank analysis which indicated a mean bearing temperature rise of 37.9°C. It must be recognized for high slip motors and special application motors this temperature may not be conservative.

TABLE 8

Typical Motor Temperature Rise Data

	Winding T	Bearing T
Shaded Pole Motor	57°C	42°C (Full Load)
Capacitor Run Motor (Totally Enclosed No Cooling Fan)	160°C	150°C (Full Load)
	77°C	66°C (Light Load)
Capacitor Run Motor (Vented -- No Cooling Fan)	95°C	82°C (Full Load
Hysteresis Motor Polyphase (fan cooled)	70°C	46°C (Full Load)
	46°C	23°C (Light Load)
Polyphase Motor (Totally Enclosed -- No Cooling)	74°C	65°C (No Load)
Inside-Out Polyphase (Fan Cooled)	32°C	50°C
High Slip Motors (Fan Cooled)	30°C	15°C (Low Slip (5%)
	50°C	90°C (80% Slip)
Polyphase Motor (Fan Cooled)	20°C	15°C

3.2.2.2 Ambient Temperature Influences on Bearing Life

It is often difficult for the user of FHP motors to obtain information regarding the bearing temperature rise over ambient temperature. It would be desirable therefore, to simplify Equation (18) and relate bearing life in terms of ambient temperature. In addition, the improvement in fit of data within a standard deviation by correcting for speed, quality and grease from Table 6 was only five percent, therefore, the simple model of Equation (10) is recommended when detailed knowledge of the motor is not available.

As a further simplification, in lieu of entering bearing temperature into regression, motor ambient temperature was utilized since this parameter was statistically significant. Equation (7) could be rewritten using ambient temperature as:

$$\text{Log life}_B = \frac{2357}{T_{amb}} - 2.534 \qquad (19)$$

When the data in the data bank was tested against this model, 63.97 of the data fell within one standard deviation. This compares with 69.85 percent using bearing temperature. Due to the possible error introduced in estimating bearing temperature, Equation (19) was selected as the general simplified model for estimating bearing life. Under normal load-speed conditions found in FHP motors, this equation will yield reasonable estimates of life. For unusual load - speed on motor designs, the more comprehensive Equation (18) should be used.

3.2.2.3 Estimating Bearing Size and Speed

When utilizing Equation (18) for predicting bearing life, it is often possible to estimate bearing size from the motor input power level. This information can be utilized to calculate the DN value of Equation (18). Table 9 is a summary of average bearing size after reviewing over two hundred different motors of various horse-power level. Bearing bore diameter is tabulated as a function of motor input power.

TABLE 9

Bearing Size Vs. Motor Input Power

Bearing Bore (D) (mm)	Power Input (watts) f = 60 Hz	f = 400 Hz
20	250-2500	
13	100- 500	200-1000
8	20- 200	150- 400
6.35	10- 50	80- 130
4.76	7- 12	60- 90
3.18	5- 8	20- 45

Motor speed can be estimated from

$$N(RPM) = \frac{120f}{P} \qquad (20)$$

where: f = line frequency, Hz

P = number of poles

3.2.2.4 Bearing Failure Data Analysis for Low Temperature

In order to establish the low temperature characteristics of the bearings, the cyclic tests of Table 10 were utilized to estimate life. Thermal cycles for all tests were not identical and the duty cycles are summarized in Table 11.

TABLE 10
BEARING LIFE TEST SUMMARY
CYCLIC TEMPERATURE TESTS

TEST TEMP TYPE	BORE DIA	SPEED HNDRS	NUMBER OF TEST UNITS		QUAL CODE	WEIBULL SLOPE	CHARACTER. LIFE
***	MM	RPM	TOTL	#FAIL	#	BETA	HOURS
AT	3.18	205	40	29	1	2.66	5533
TT	4.76	110	5	5	1	0.71	9328
TT	4.76	110	5	5	1	1.25	10604
AT	4.76	205	5	3	1	0.94	4822
TC	4.76	225	5	4	1	2.36	6653
AT	3.18	200	30	3	1	1.37	17094
AT	3.18	200	10	8	1	2.79	13931
TC	6.35	90	6	6	1	3.80	1758
TC	6.35	90	4	4	1	2.09	771
TC	6.35	90	3	3	1	6.03	1879
TC	6.35	90	3	3	1	2.80	1516
AT	6.35	105	5	5	1	1.10	1167
AT	6.35	105	5	5	1	1.16	2854
AT	6.35	110	10	10	1	2.89	2738
AT	6.35	60	8	8	1	0.73	1902
TC	8.10	106	3	3	1	2.32	13965
AT	3.18	205	13	4	1	3.28	11966
AT	3.18	205	11	9	1	2.79	19518

TABLE 11

Thermal Cycling Tests

Duty Cycles

Test Number	Thermal Cycle
1106	8 hrs. off 4 hrs. at -42°C to +55° at 3° per min.
656, 622	39 hrs. at -54°C 241 hrs. at +93°C remaining at +22°C
1384, 1324, 1456 1377, 1374	2 hrs. off 2 hrs. at -55°C to +100°C
1492	150 hrs. at +25°C 50 hrs. at -55°C 100 hrs. at +125°C 200 hrs. at +75°C
700 713, 728, 313	100 hrs. at +25°C 50 hrs. at -55°C 250 hrs. at +55°C 100 hrs. at +85°C
1533, 1544	2 hrs. off 1 hr. at -65°C 1.5 hrs. at -65°C to 71°C 1.5 hrs. at 71°C
1556, 1517	1.5 hrs. off 2.5 hrs. at -55°C 1.5 hrs. at -55°C to +72°C 1.0 hr. at +72°C
765	15 hrs. at +72°C 1 hr. at +72°C to 23°C 7 hrs. at 23°C

From the characteristic life of Table 10 and the duty cycles of Table 11, the number of thermal cycles prior to failure could be determined. The life at bearing temperatures of 50°C or greater could be computed using Equation (18) previously developed. The characteristic life at low temperature could then be solved from Equation (21).

$$\text{No. of cycles (N)} \left(\frac{\text{hrs. at } T_1}{\text{Life at } T_1} + \frac{\text{hrs. at } T_2}{\text{Life at } T_2} + - - \right) = 1 \qquad (21)$$

TABLE 12

Low Temperature Bearing Life

Test Number	T_B (°C)	Characteristic Life (Hrs)
1106	+10	304
656	-21	46
622	-21	50
1384	+28.5	2041
1496	-17	1112
1324	+22.5	5432
1456	+22.5	4203
700	-35	382
713	-35	80
728	-35	406
913	-35	323
1533	-45	220
1544	-45	635
1556	+10	833
1517	+10	574
765	+20	4628
1377	+28.5	4225
1374	+28.5	9315

The resulting low temperature characteristic life determined from this equation is tabulated in Table 12. The points are also plotted on log-reciprocal temperature paper in Figure 3. The solid line is a fit of data between 10°C and 30°C. The equation of the line is:

$$\text{Log life}_B = \frac{-4760}{T_B} + 19.7 \qquad (22)$$

Considerable scatter in the data exists below 10°C and it was decided to bound the minimum life at 300 hours. Life at low temperatures therefore, could be expressed in the form

$$\text{Characteristic life}_B = 10^{\left[\frac{-4760}{T_B} + 19.7\right]} + 300 \text{ hrs.} \qquad (22)$$

This equation is shown as the dotted line in Figure 3.

From previous discussions (Section 3.2.1.2) regarding low temperature life, it is anticipated that the type grease and speed will have considerable influence on the prediction model at low temperature. It is suspected that the major influence is to shift the solid line of Figure 3 to the left or right and the slope of the line will not be significantly changed.

Since an expression was derived to predict life of the bearing as a function of ambient temperature in the higher temperature region, it is desirable to obtain a similar expression for low temperature life as a function of ambient temperature. Utilizing ambient temperature, the equation derived for life was:

$$\text{Log life}_B = \frac{-4500}{T_{amb}} + 20 \qquad (23)$$

Setting the lower life bound of 300 hours Equation (23) was modified to:

$$\text{Life}_B = 10^{\left[\frac{-4500}{T_{amb}} + 20\right]} + 300 \text{ hours} \qquad (24)$$

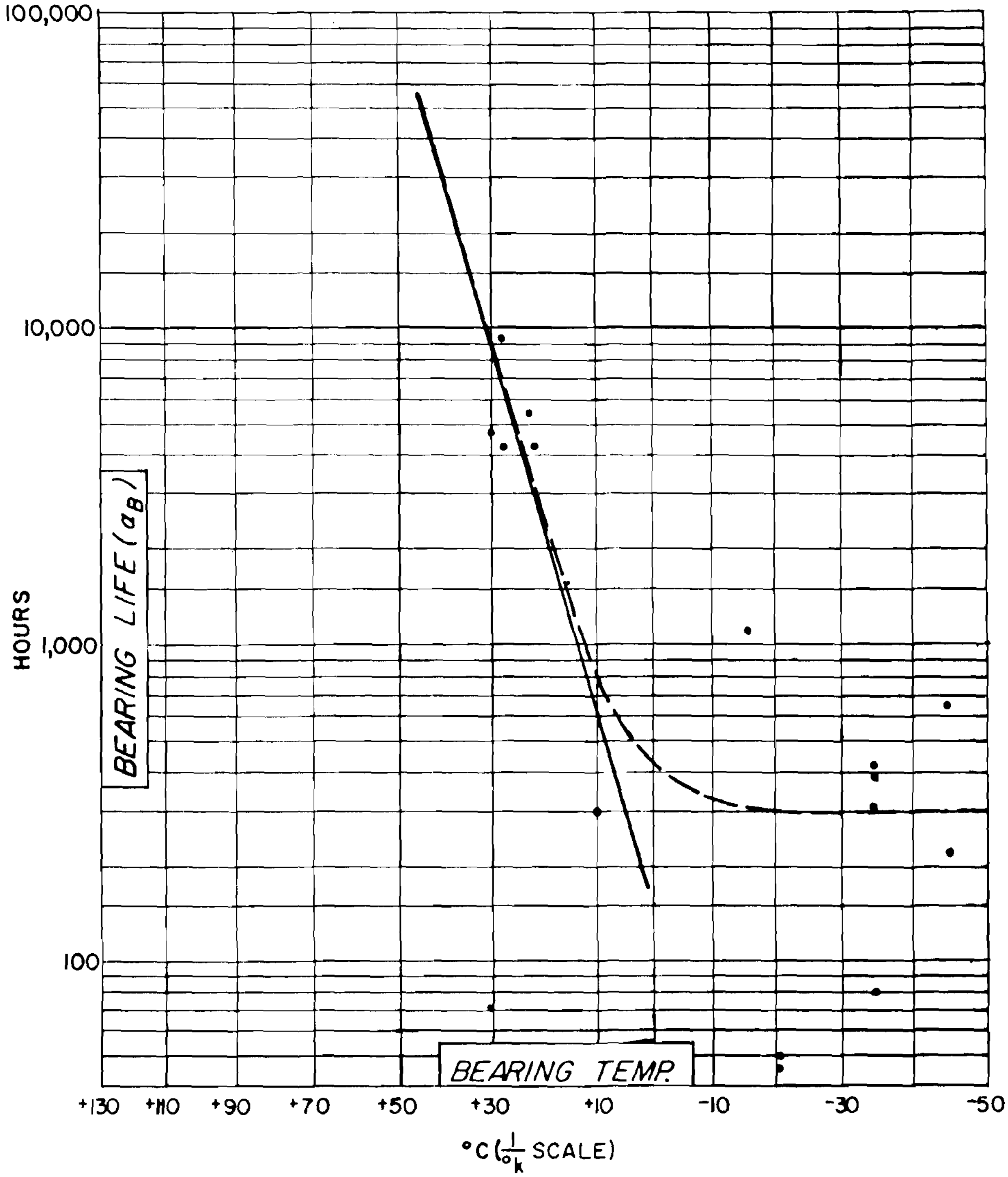


Figure 3

Bearing Life at Low Temperature

Combining the low temperature and high temperature models, Figure 3 can be modified to include the bearing life model over the entire temperature range. The combined life is illustrated in Figure 4.

It is not expected that the transition of life from an increasing to a decreasing function is as abrupt as the solid lines in Figure 4. Therefore, the equations for high temperature life and low temperature life were combined in the form of Equation (25) and shown as the solid and dotted lines of Figure 4.

$$\frac{1}{\text{Life}} = \frac{1}{\text{Life (High Temp)}} + \frac{1}{\text{Life (Low Temp)}} \qquad (25)$$

Two models have been developed for predicting bearing life. The more comprehensive, Equation (18), requires considerable knowledge regarding the motor. The second, Equation (19), has a prediction estimate accuracy 11 percent lower than Equation (18); its use requires knowledge of the ambient temperature only. These two equations can be modified to include low temperature influences using Equation (25) with the following overall bearing model:

$$\frac{1}{\text{Life}} = \frac{1}{10^{\left[\frac{2342}{T_B} + q - 4.32DNX \times 10^{-6} - K_g - .001N\left(\frac{W}{S.P.}\right)^{1.5}\right]}} + \frac{1}{10^{\left[\frac{-4760}{T_B} + 19.7\right]} + 300} \qquad (26)$$

$$\frac{1}{\text{Life}} = \frac{1}{10^{\left[\frac{2357}{T_{amb}} - 2.537\right]}} + \frac{1}{300 + 10^{\left[\frac{-4500}{T_{amb}} + 20\right]}} \qquad (27)$$

The resulting life is the Weibull characteristic life or 63.2 percentile point of the distribution. The probability is that 63.2 percent of the population will have failed at this life.

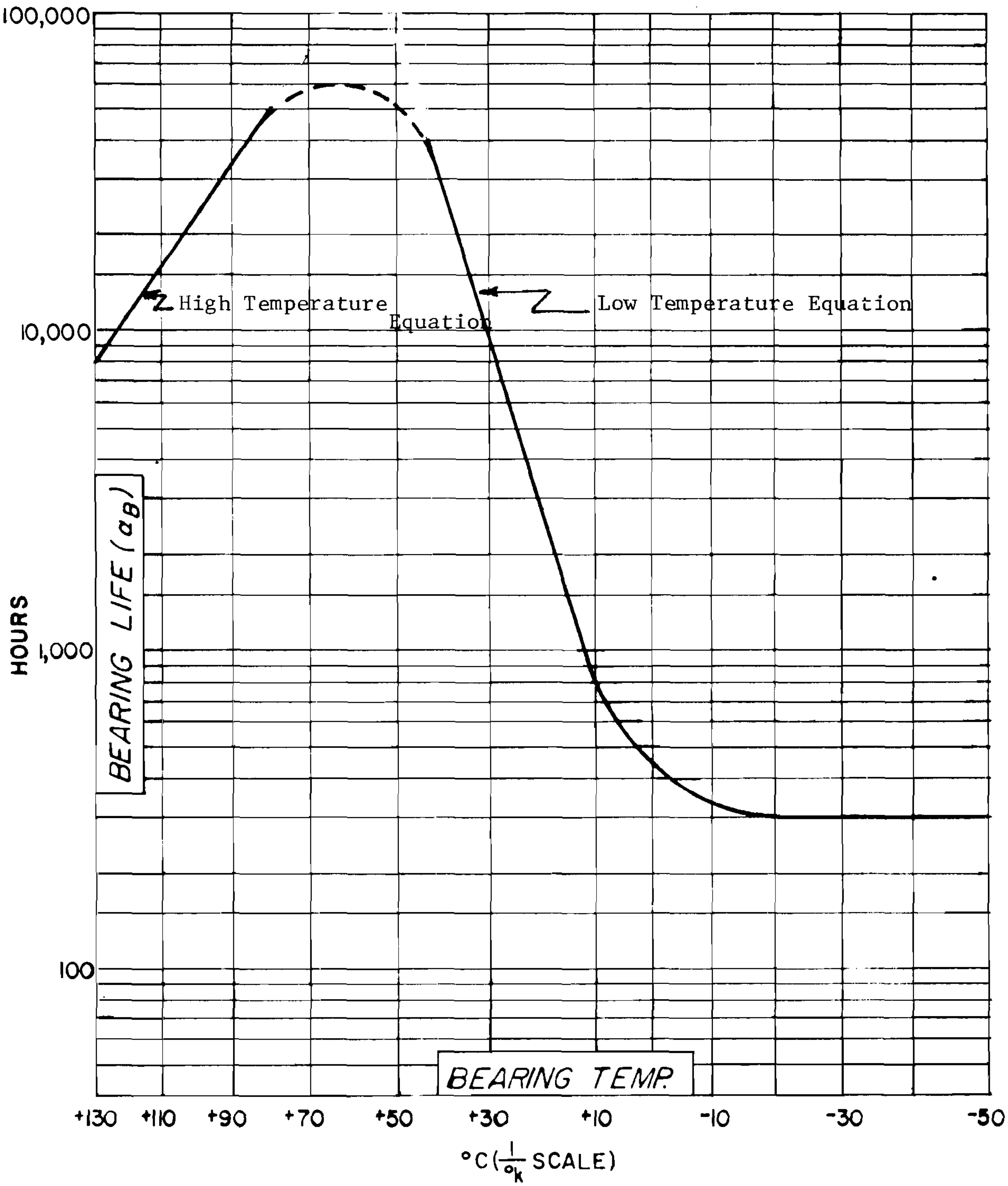


Figure 4

Bearing Life over Temperature Range

3.2.2.5 Weibull Slope for Bearing Life

Equations (26) and (27) predict the bearing Weibull characteristic life (α_B). In order to determine the life at any other percentile, it is necessary to establish the Weibull Slope (β_B). In reliability prediction for electronic components, the mean life is of interest in order to determine system failure ratesfor constant failure rate components. Although bearings are an increasing failure rate component, a reasonable estimate of the mean life is the 50 percentile point of the Weibull distribution. The average failure rate at this percentile is suggested as a means of comparing bearing failure rate with constant failure rate components.

In addition to establishing bearing failure rates, it is also desirable on occasion to determine the 10 percentile point often referred to as the B_{10} life in bearing analysis. In order to determine life at any percentile, Equation (18) or (19) can be modified with knowledge of the Weibull slope (β_B) as follows:

$$\text{Log life} + \frac{1}{\beta_B} \text{ Log}\left(\ln \frac{1}{1-F(t)}\right) = \frac{A}{T} + B \qquad (28)$$

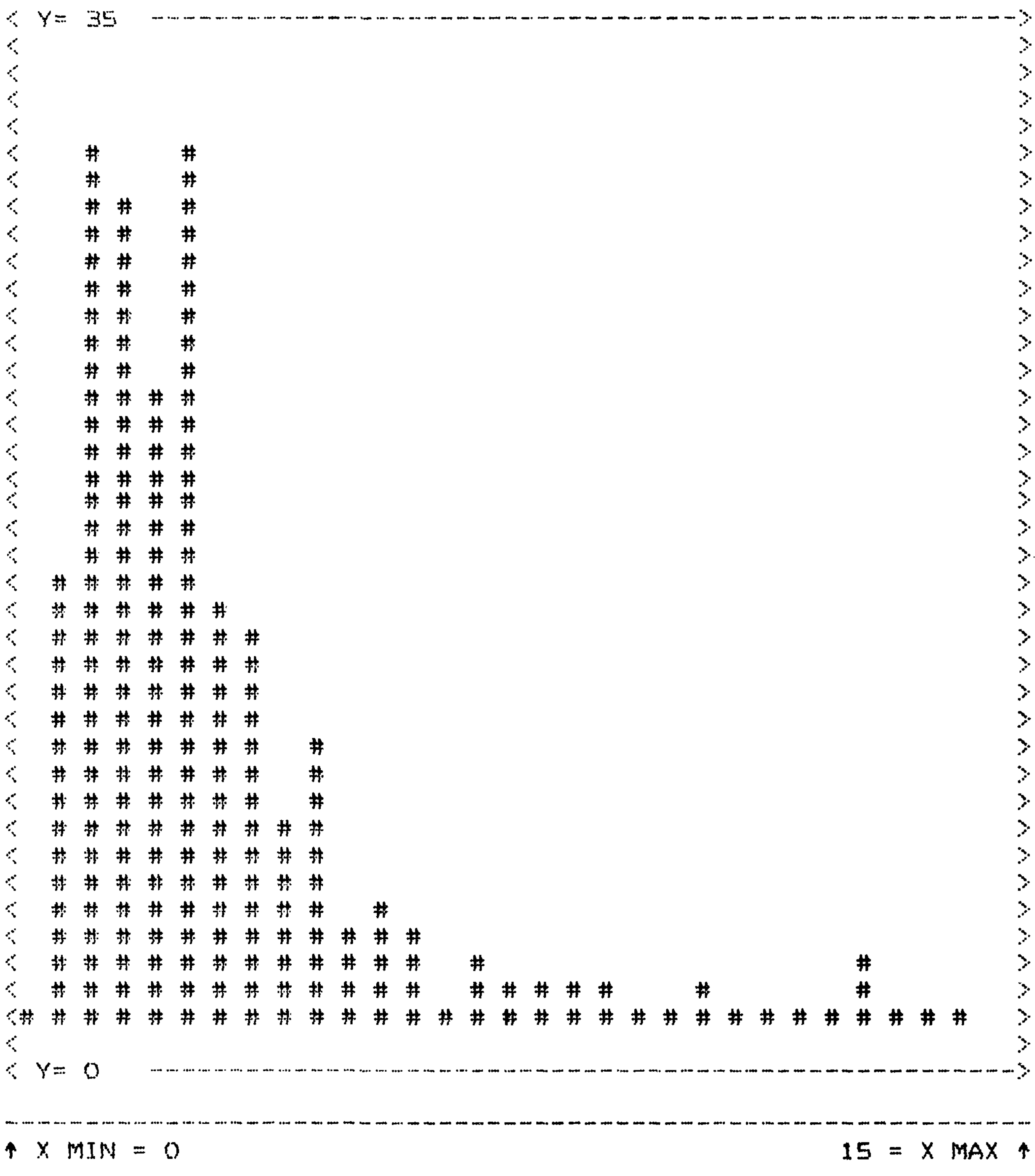
where $F(t)$ = failure percentile

The distribution and mean of the Weibull slope from Tables 1, 2, and 3 were used to establish the average slope for use in Equation (28). Figure 5 is a histogram of the distribution of the Weibull slope of bearing failures. The mean of the distriburion is 2.8789 with a standard deviation of 1.99. The mean value of 2.878 was selected as Weibull slope β for bearing failures.

3.2.2.6 Bearing Life -- Grease Constants

In addition to the three basic greases, analysis was conducted on 21 additional greases to establish the grease constant K_g. The values of grease constant ranged from 1.37 to 2.35 with a mean value of 1.825. A number of these constants were developed from a small test population and therefore a question

WEIBULL SLOPES
HISTOGRAM



BAR WIDTHS = .5

BEARING TEST DATA -- ALL GREASES

FIG. 5

arises as to the reliability of the estimates. The population of test units for three greases was sufficiently large to include reporting the values of the constants.

The grease constants were developed from Equation (18); i.e.

$$\text{Log life} + \frac{1}{2.878} \log\left(\ln \frac{1}{1-F(t)}\right) = \frac{2342}{T_B} + q - 4.32\ DN \times 10^{-6}$$

$$- K_g - .001N \left[\frac{W}{SP}\right]^{1.5}$$

The resulting K_g values for all greases can be summarized as follows:

Grease	Oil	Thickener	Mil Spec.	Max. Temp. °C	K_g
1	diester	Sodium and solid Lubricant	MIL-G-3278A	170	1.35
2	diester	Lithium	MIL-G-3278A	120	1.55
3	Silicone	Lithium	-----	150	1.74
4	Mineral	Sodium	MIL-G-18709A	150	1.41
5	Silicone	Lithium	MIL-L-15719A	177	1.81
6	Syn. Hydrocarbon	Non-soap	MIL-G-81322	170	1.74

3.3 Motor Winding Reliability Model

Winding failures are the second dominant failure mode of FHP motors. Table 4 summarizes winding failures as a function of temperature. By conducting multivariable regression analysis using variables associated with windings, a reliability model was developed for prediction of winding life. The following sections discuss the literature survey conducted on windings as well as development of the predictive model.

3.3.1 Literature Review of Electrical Failures

There is general agreement between investigators of electrical insulation life that chemical changes of the insulation are dominant factors in insulation deterioration. It is well known that chemical reaction rates vary with the temperature according to the Arrhenius Equation (29):

$$\text{Insulation life} = \frac{1}{\text{reaction rate}} = A\, e^{B/T} \qquad (29)$$

$$\text{or Log life} = \log A + \frac{B}{T}$$

This is the general form that is used to express insulation life. In reviewing literature data, life was transposed into the form of Equation (29).

Most testing done on insulation systems are not conducted directly in the motor but are run in test units called motorettes. To accelerate insulation degradation, tests usually include cycles of 100 percent humidity and vibration. Data presented in Reference 10 indicate that the humidity cycling introduces between 4 and 5 to 1 reduction in life. Beebe (11) suggests that actual winding life in motors exceeds motorette life by 4 to 1. This implies that humidity is the dominant cause of life-reduction in motorette tests.

Table 13 summarizes the test data and results reported in the literature:

TABLE 13

Literature Review of Temperature Influence on Life

Log Avg. Life	Weibull Slope	Insulation Class	Type Unit	Humidity Included	Reference
$\frac{5948}{T} - 10.7$	-	B	Motorette	No	12
$\frac{7330}{T} - 10.38$	-	H	Motorette	No	12
$\frac{3275}{T} - 4.615$	-	A	Motor	No	13

(continued)

Table 13 (continued from previous page)

Log Avg. Life	Weibull Slope	Insulation Class	Type Unit	Humidity Included	Reference
$\frac{4282}{T} - 6.95$	-	A	Motorette	No	13
$\frac{4509}{T} - 8.62$	-	A	Motorette	Yes	13
$\frac{7474}{T} - 11.97$	-	H	Motorette	Yes	13
$\frac{7583}{T} - 12.2$	-	H	Motorette	Yes	13
$\frac{7175}{T} - 10.0$	-	H	Motor	Yes	14
$\frac{6311}{T} - 11.92$	-	A	Motorette	Yes	14
$\frac{4661}{T} - 7.02$	-	B	Motorette	Yes	14
$\frac{3474}{T} - 5.03$	3.0	A	Motor	Yes	15
$\frac{6344}{T} - 10.0$	1.6	B	Motor	Yes	15
$\frac{3560}{T} - 5.24$	-	A	Motorette	Yes	10
$\frac{5347}{T} - 8.9$	-	A	Motorette	No	16
$\frac{3733}{T} - 7.02$	-	A	Motor	Yes	16
$\frac{4054}{T} - 6.06$	-	A	Motor	Yes	17
$\frac{3433}{T} - 4.96$	-	A	Motor	Yes	17
$\frac{3945}{T} - 6.17$	4.5	A	Motorette	Yes	18
$\frac{4445}{T} - 7.398$	-	A	Motorette	Yes	18

(continued)

Table 13 (continued from previous page)

Log Avg. Life	Weibull Slope	Insulation Class	Type Unit	Humidity Included	Reference
$\frac{4406}{T} - 7.415$	6.8	A	Motorette	Yes	18
$\frac{5003}{T} - 8.996$	-	A	Motorette	Yes	11
$\frac{4551}{T} - 7.21$	-	B	Motorette	Yes	11
$\frac{6954}{T} - 12.17$	-	F	Motorette	Yes	11
$\frac{4994}{T} - 8.71$	-	A	Motorette	Yes	19
$\frac{4785}{T} - 8.575$	-	A	Motorette	Yes	19
$\frac{5123}{T} - 8.79$	-	A	Motor	No	20

A considerable spread exists in literature regarding the experimental life of windings. The reported reasons for the variation is due to difficulty in controlling test cycles such as humidity, variations in the insulating materials tested in each winding class, different voltages applied to windings and variations in interpretation of failure when a failure occurs.

One further concern noted in the available life data is the temperature range used to establish life characteristics. The NEMA specified winding classes A, B, F and H correspond in general to IEEE classes 105°C, 130°C, 150°C and 180°C. These temperatures are the recommended operational limits for each insulation class to insure reasonable lives in the range of 20,000 to 40,000 hours. In order to accelerate testing, test temperatures are selected above the minimum recommended rated temperatures. This tends to force the failure mode to one of chemical degradation of the insulation. At temperatures below the recommended limits, other environmental and mechanical considerations can influence life. This includes factors such as dirt, oil, chemicals, oxygen and moisture as well as thermal stresses resulting from thermal cycling. In addition to environmental factors, the actual quality in manufacture of production motors, such as nicks in the insulation during winding, or improperly seated wedges in slots can lead to early failures. Quality in the very small fractional horsepower motors become even more important because of physical size. All of these factors tend to induce occasional failures at low temperatures which would not be predicted by the Arrhenius equation. It is anticipated therefore, that data from the failure bank will contain some early failures and will not be completely compatible with literature predictions.

The following equations for life may be derived for the average slope and intercept from the previous literature tabulation of winding life:

Class A ($105^{o}C$) $\text{Log life} = 4400/T - 7.49$ (30)

Class B ($130^{o}C$) $\text{Log life} = 5400/T - 8.78$ (31)

Class F ($150^{o}C$) $\text{Log life} = 7000/T - 12.28$ (32)

Class H ($180^{o}C$) $\text{Log life} = 7800/T - 11.88$ (33)

These equations indicate motor insulation life including humidity cycles. Lives of 3 to 4 times these values can be anticipated when applied to actual motors. As previously indicated, other winding failures may be anticipated in motors due to quality factors and environmental influences. This is especially true in the case of fractional horsepower motors since the small wire sizes can readily be damaged or over stressed during winding. To verify this condition, the failure data bank was reviewed for winding failure for motors with stator lamination diameters from 1.5 to 3.0 inches. The data was reviewed considering the number of winding failures versus the total number of failures. (The remaining failures were all bearing failures.) A similar review was conducted on motors with stator laminations of one inch diameter. Table 14 summarizes these results.

For both motor sizes, it is noted that, the failure percentage of windings below a winding temperature of $160^{o}C$ ranges from 3.5 to 11.1 percent. The average failure percentage of windings in this range is approximately 8%.

This is much higher than predictions from the literature would imply for this temperature range due to degradation of insulation.

TABLE 14

MOTOR WINDING FAILURES

(1.5" - 3.0" Stator O.D.) (Class F & H Insulation)

Winding Temperature °C (By resistance)	Total Tested	Winding Failures	Percent Winding Failures
75	31	2	6.5
114	172	6	3.5
130	137	15	10.3
160	208	19	9.13
185	163	3	1.84
(1.0" Stator O.D.) (Class H Insulation)			
75	27	3	11.1
100	75	5	6.7
122	98	8	8.2
175	392	69	17.6
190	10	5	50.0

The second point noted from these tables is that the smaller motors have higher failure percentages of windings at the higher temperatures, than the larger motors. In fact, windings become a small percentage of failures in the larger motors at high temperature. At these higher temperatures, bearing life drops significantly; therefore, it is expected that winding failures would not be too important. However, in the small motors, the higher temperatures accelerate winding failure. The larger motors are wound with magnet wire of 0.005 inches and larger while the small motors use wire of 0.0035 inches and smaller. Difficulty in maintaining uniform stress during winding, the compactness of winding and the small turn radius necessary for the small wire are considered primary causes for the premature failure at high temperature.

In predicting motor life, it appears that consideration must be given to motor size as well as to the random failures encountered in the low temperature range. During analysis of the failure bank data these two areas were considered.

3.3.2 Regression Analysis of Winding Failures

Table 14 outlines the results obtained by reviewing the number of winding failures in overall motor failures. It was pointed out that winding failures represented between 3.5 and 10 percent of the total failures at winding temperatures below 160°C. If we utilize literature predictions for winding life, the results lead to lives orders of magnitude greater than bearing life. This implies that winding failures would not be encountered in motors using grease-pack ball bearings. Table 14, however, does not verify this condition. Therefore, it is anticipated that these failures are not due entirely to insulation aging and may be more random in nature.

Table 4 is the summary of winding failures from the failure data bank. Inspection of the data indicates that the significant portion of failures occur at ambient temperatures of 125°C and above. This also follows the trend of data presented in Table 14 for small diameter (1.0 inch) motors. Although not indicated in Table 4, all failures at 125°C and above are indeed small diameter motors. Regression analysis of life versus temperature therefore will be dominated by the lower lives at higher temperature. A linear regression was conducted of the data of Table 4 entering the following variables temperature; number of poles; voltage; speed; winding size; insulation type, and motor type. The analysis indicated that temperature and motor quality represented the significant variables influencing winding life. The results of the analysis indicated winding life could be expressed as:

$$\text{Log life} = \frac{4333}{T_w} - 3.52 \pm q \qquad (34)$$

where q = quality code

commercial quality = -.13

Mil-spec. quality = +.13

In comparing these results with Equations (32) and (33) for Class F and H insulation, there is very poor correlation. It would appear that the lower lives of the small motors dominate the prediction model of Equation (34).

To determine the influence of the high ambient temperature failures, winding life of units operating at an ambient temperature below 125°C was plotted as a function of temperature on Figure 6. Although the regression line is drawn from only five points it does indicate a temperature slope approaching that of bearing life, i.e.:

$$\text{Log life (winding)} = \frac{2395}{T_{amb}} - 2.064 \qquad (35)$$

$$\text{Log life (bearings)} = \frac{2357}{T_{amb}} - 2.534 \qquad (19)$$

If the lower temperature winding failures are random in nature, it would be anticipated that the Weibull slope would be 1. Therefore, the mean and standard deviation of the Weibull slope for all winding failures as well as low temperature winding failures was conducted as tabulated in Table 15.

TABLE 15

Winding Failure Weibull Slope

Mean Values

	Mean	Std. Dev.
All Winding Failures	1.961	1.195
At or Above 125°C Failures	2.329	1.250
Less than 125°C Failures	1.085	.716
(Bearing Failures)	(2.878)	(1.997)

From the Weibull slopes of Table 15, winding failures below 125°C ambient have a mean slope approaching 1 indicating random type failures with a constant failure rate. Failures of the 1.0 inch diameter motors at 125°C ambient have a mean Weibull slope of 2.329 indicative of wear out type failures. The influence of life on temperature of data in the data bank therefore,

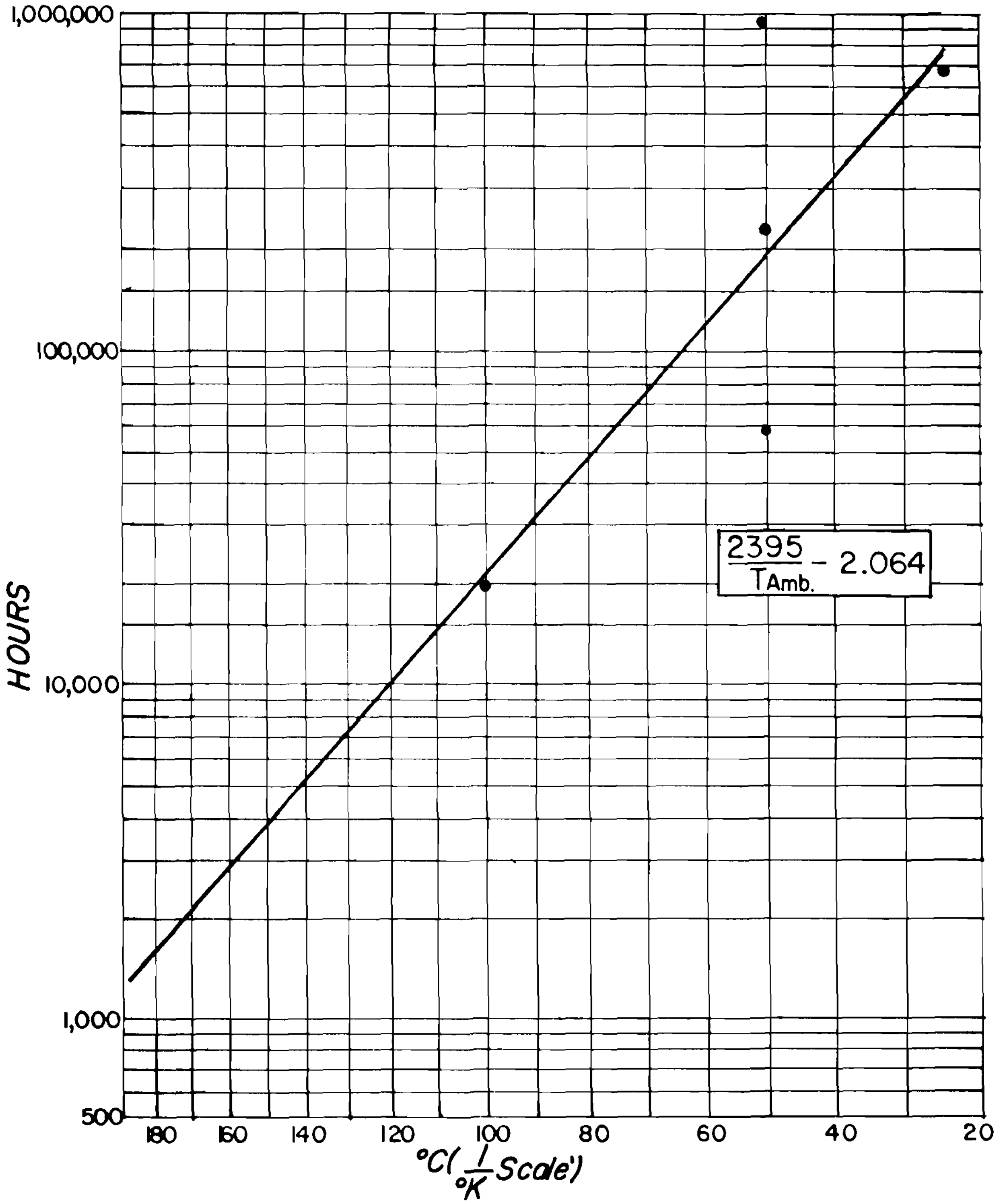


Figure 6

Winding Life
Versus
Ambient Temperature

appears to be influenced by two different failure modes. Equation (34) therefore, does not appear to be a reliable model for predicting temperature-life characteristics. The wear out Equation (35) for motors larger than 1 inch diameter appears to be a more realistic model for winding influences. For these motors, the percentage of winding failures versus bearing failures appear to be almost constant over the temperature range. If we make the assumption that winding failures are a constant percentage of total failures over the temperature range, Equation (35) can be modified to accommodate this relationship with the slope of log life versus temperature the same for both bearings and windings.

$$\text{Log life winding} = \frac{2395}{373} - 2.064 = 4.357$$

Characteristic life = 22746 hours

Weibull Slope = 1.0

$$\text{Log life bearings} = \frac{2357}{373} - 2.534 = 3.785$$

Characteristic life = 6096 hours

Weibull slope = 2.88

Figure 7 illustrates a Weibull plot of bearing life of the equation above and winding life at a slope of 2.88 and 1.0, respectively. The curves cross at the 11 percentile point indicating that approximately 11 percent of the failures will be electrical as a competing risk failure model. This is slightly higher than the average failure percent of 8.0 from Table 15. The plot does illustrate the method to be used for predicting winding life. That is, the average 8 percent winding failures is used from Table 15 as typical of the number of winding failures of motors versus bearings. A Weibull slope of 1 is used for winding failures. The winding life and bearing life at the 8 percentile point are equal; i.e.

$$\text{Log life}_w + \frac{1}{\beta_w}\left(\log \ln \frac{1}{1-F(t)}\right) = \text{Log life}_B + \frac{1}{\beta_B} \log \ln\left(\frac{1}{1-F(t)}\right)$$

$$\text{Log life}_w = \text{Log life}_B + \frac{1}{2.88} \log\left(\ln \frac{1}{1-.08}\right) - \frac{1}{1} \log\left(\ln \frac{1}{1-.08}\right)$$

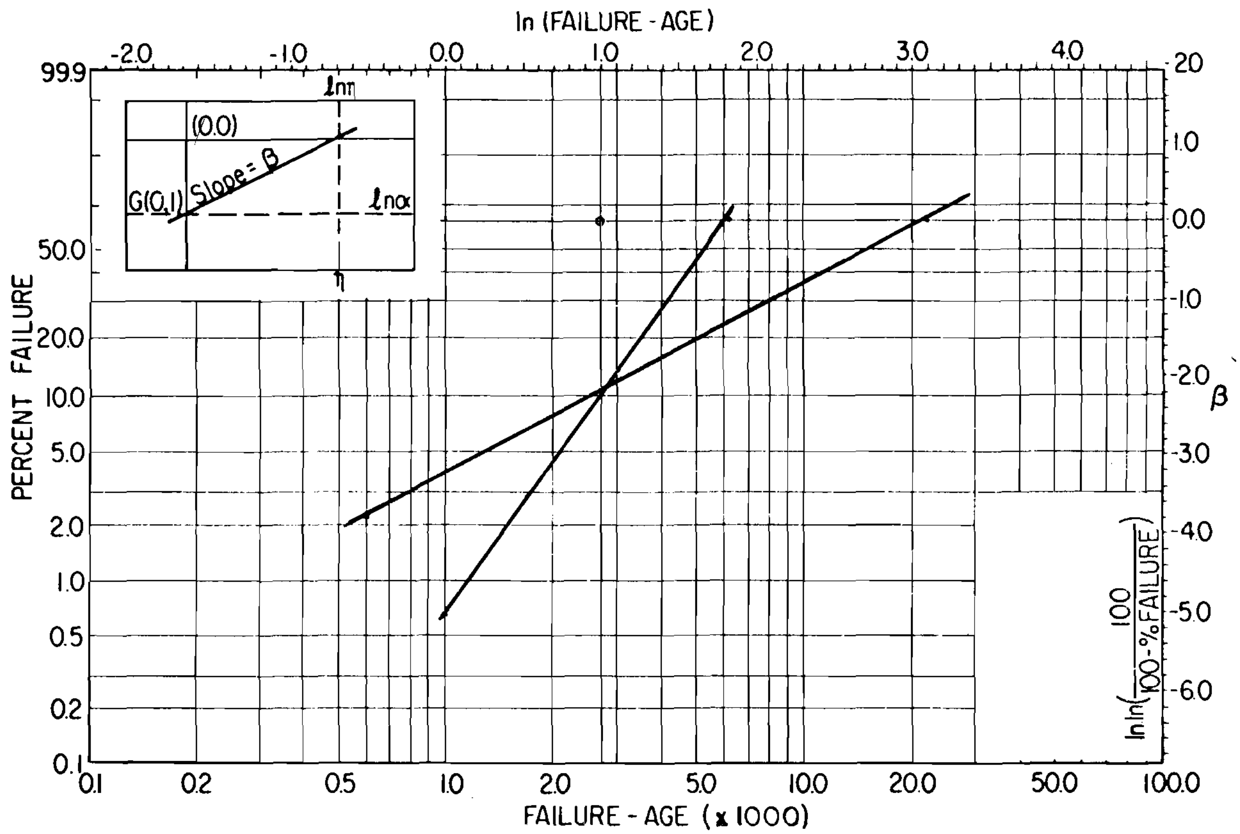


Figure 7

Winding Life
Versus
Bearing Life at 100°C Ambient

$$\text{Log life}_w = \text{Log life}_B + .704$$

$$\text{Log life}_w = \frac{2357}{T_{amb}} - 2.534 + .704$$

$$\text{Log life}_w = \frac{2357}{T_{amb}} - 1.83 \qquad (36)$$

In a similar manner, the life of the winding versus winding temperature was developed as

$$\text{Log life}_w = \frac{2342}{T_w} - 1.206 + q \qquad (37)$$

where: q is defined from Equation (34)

These two equations provide the analytical model for random failures encountered in windings.

Winding life at ambient temperatures of 125^{o}C for motors with stator diameters of 1.0 inch and smaller have reduced life from Equation (36) or (37). A regression analysis was conducted on only small motors at temperatures above 125^{o}C ambient with the following results:

$$\text{Log life}_w = \frac{4500}{T_w} - 6.328 \qquad (38)$$

$$\text{Log life}_w = \frac{4900}{T_{amb}} - 8.2 \qquad (39)$$

Two equations were developed -- one using ambient temperature the other winding temperature. It was stipulated in the sections discussing bearing life that it is often difficult to find the bearing temperature without measurements or information furnished by the manufacturer. The same problem exists with predicting winding temperature. Table 8 provides a summary of typical motor temperature rise data. Because of the broad range of temperature rise that could be encountered, a mean rise of 36^{o}C (see Figure 2) is recommended when better information is unavailable. To provide further simplication, it is suggested that ambient temperature be used for the winding failure model when actual winding temperature rise is uncertain.

To retain the simplicity of the winding failure model using ambient temperature, it is suggested that the influence of quality and motor size be disregarded for this model. The more complex model utilizing winding temperature which necessitates more detailed information regarding motor characteristics would include quality and motor size. Equations (36) through (39) would reduce to:

a) Winding life using ambient temperature:

$$\text{Log life}_w + \text{Log}\left(\ln \frac{1}{1-F(t)}\right) = \frac{2357}{T_{amb}} - 1.83 \qquad (40)$$

b) Winding life using winding temperature:

$$\text{Log life}_w + \log\left(\ln \frac{1}{1-F(t)}\right) = \frac{2342}{T_w} - 1.206 + q_w \qquad (41)$$

where: q_w = -.13 commercial quality
= +.13 Mil. spec. quality

c) Winding life using winding temperature and motors with stator O.D. 2.54 cm or less:

$$\frac{1}{\alpha_w} = \frac{1}{10^{\left[\frac{2342}{T_w} - 1.206 + q_w\right]}} + \frac{1}{10^{\left[\frac{4500}{T_w} - 6.328 + q_w\right]}}$$

where α_w = characteristic life (hrs) (42)

Equations (40) and (41) present the failures at any percentile (F(t)) while Equation (42) is the predicted life for the characteristic life only.

The multiplier for life at other percentiles is:

At winding temperatures of 160^{o}C and greater:

$$\text{life} = \alpha_w \left(\ln \frac{1}{1-F(t)} \right)^{2.329} \tag{43}$$

At winding temperatures of 159^{o}C or lower:

$$\text{life} = \alpha_w \left(\ln \frac{1}{1-F(t)} \right) \tag{44}$$

It has been suggested that ambient temperature neglecting quality code (Equation 40) be used for predicting winding life in the absence of detailed knowledge regarding the motor. Neglecting the quality code introduces a 35 percent error in determining winding life. This is equivalent to an error of 8^{o}C in estimating temperature. Winding temperature rise using Equation (40) is an average rise of 36^{o}C as shown in Figure 2. In many applications, predicting ambient temperature duty cycle introduces errors in the range of 10^{o}C or greater. The quality code therefore, is less significant in the introduction of errors than is the temperature influence and the use of a nominal quality value is proposed to maintain the ease in use of Equation (40). If it is known that the motor runs considerably hotter or cooler than 36^{o}C, Equation (41) should be applied.

Neglecting small motors of 1 inch (2.54 cm) stator lamination diameter and smaller introduced error only when ambient temperatures exceed 120^{o}C. The motor size and ambient temperature represents a very small portion of the total motor population. Therefore, neglecting small motors at high temperature is felt warranted for a generalized FHP winding failure prediction model.

It should be pointed out that in the development of failure prediction models for both bearings and windings, the simplified equations are developed for use in MIL-HDBK-217B. The approach is to provide an ultimate FHP motor failure model that requires the minimal of information from the user to estimate motor life. It is felt that the simplified equations can perform that function if the user is aware that a more comprehensive approach is available for unusual applications of temperature, speed, load or environment.

3.3.3 Influence of Altitude on Bearing and Winding Temperature

A number of individual tests in the failure data bank were run at pressures simulating altitudes between 50,000 and 85,000 feet. Bearing temperature rise at these altitudes were assumed to be 40°C. Since each test was a single test point and several tests were run at each temperature, the average life at each ambient temperature plus 40°C was plotted on Figure 8. Also plotted was the predicted life from:

$$\text{Log life} + \frac{1}{2.88} \log \ln\left(\frac{1}{1-F(t)}\right) = \frac{2342}{T_B} - 1.91 \qquad (10)$$

for both the characteristic life (F(t) - 0.632) and the B-50 or fifty percent failure (F(t) = 0.5). The B-50 life is more suitable a comparison with average life. It will be noted from Figure 8 that altitude operation of 50,000 feet introduces an underestimate of winding temperature of approximately 95°C. At a 125°C assumed bearing temperature, the reduction in life can be seen to be 10 to 1.

Unfortunately, data is not available at altitudes below 50,000 feet to provide an indication of the influence of life versus altitutde. Since the life influence is so significant, a derating factor should be applied to life with operation at altitude.

Veinott (21) does provide some indication of the influence of altitude on motor temperature. Figure 9 taken from Reference 21 indicates the temperature rise characteristics of internally fan cooled and non-fan cooled totally enclosed motors versus altitutde. At altitude levels of 50,000 feet, the temperature rise of fan cooled motors are approximately 5 times the sea level temperature rise while in totally enclosed motors the rise is 2 to 1 that at sea level. This explains the higher bearing temperature rises associated with Figure 8. These numbers however, are somewhat misleading since the ambient tempera-

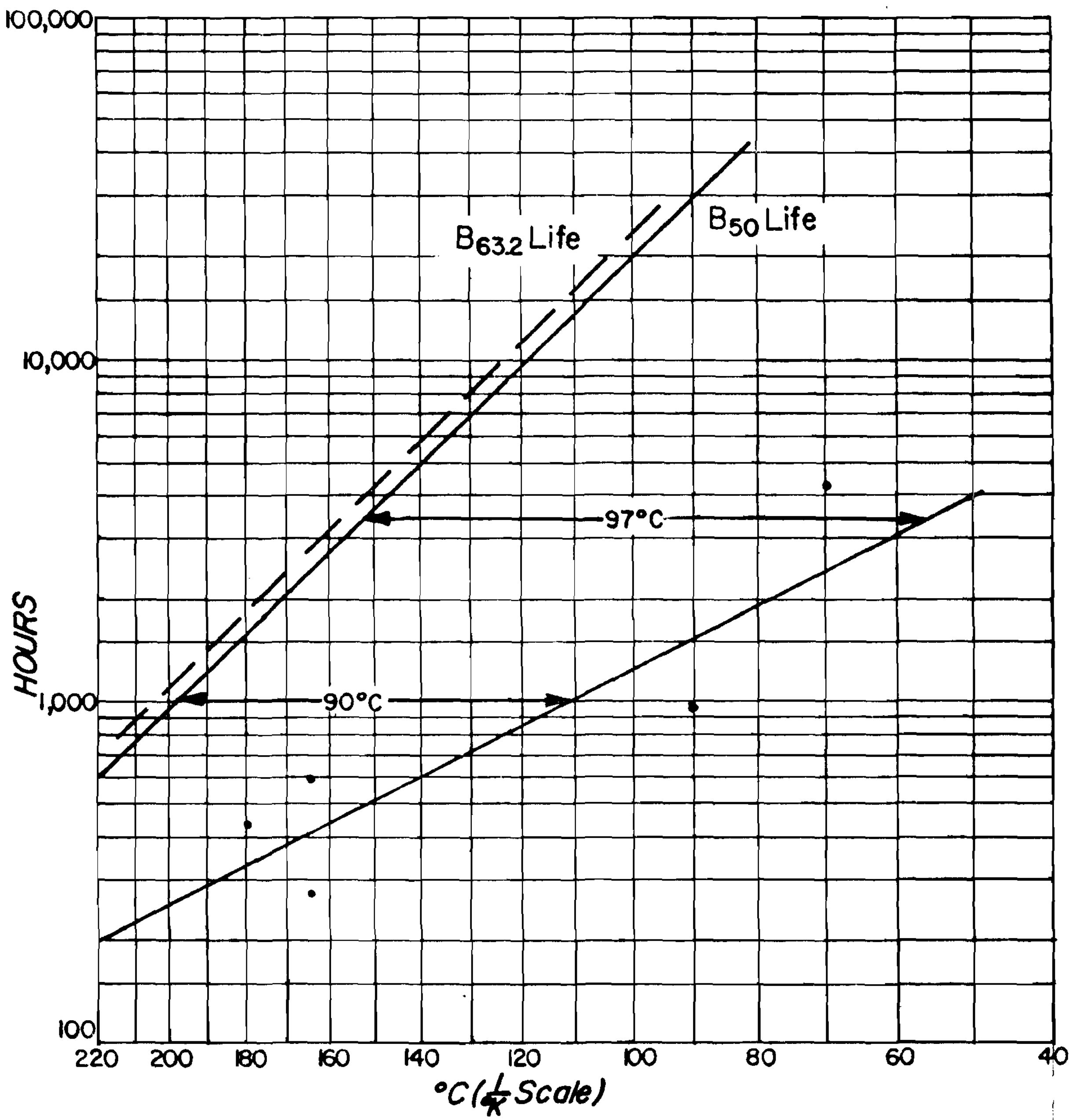


Figure 8

Influence of Altitude on Life

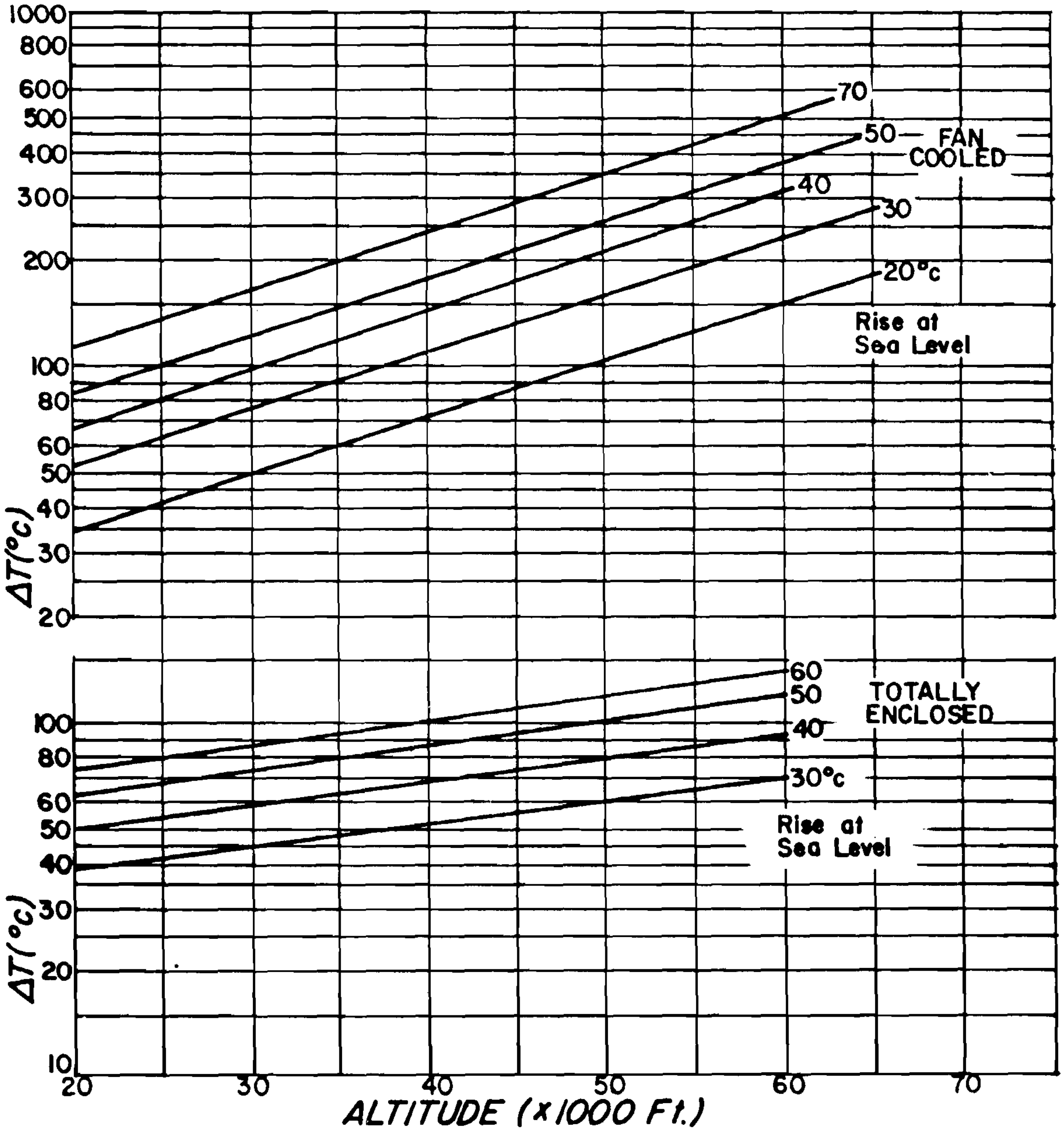


Figure 9

Temperature Rise Characteristics
Internally Fan Cooled and Non-Fan Cooled
Totally Enclosed Motors
Versus Altitude

ture also decreases with altitude. At 50,000 feet for standard conditions, the ambient temperature is -55°C. For a fan-cooled motor having a 50°C rise at sea level, winding temperatures can reach 200°C. At sea level, the winding temperature would be 65°C. The influence of 50,000 feet altitude therefore would be 3 to 1 over sea level temperature. In a totally enclosed motor, at 50,000 feet, the winding temperature would reach 50°C compared to 65°C temperature at sea level. This simply points out that for a fan-cooled motor with a sea level winding temperature rise, altitude will not result in higher winding temperatures below 35,000 feet. Above this altitude rapid reduction in life can be anticipated. For totally enclosed motors, altitudes to 60,000 feet will not influence life as compared to sea level. In each case it is assumed the ambient temperature follows the standard temperature-altitude characteristics.

From these results, the influence of altitude on life for a fan-cooled motor may be tabulated based on a 50 percent reduction in life for every 10°C increase in temperature. Table 16 is a tabulation of life multipliers versus altitude based on Army maximum condition temperatures.

TABLE 16 (Reference 21)

Influence of Altitude on Motor Life for Fan-Cooled Motors

Altitude (thousands of feet)	Life Multipliers for Sea Level Motor Temperature Rise				
	20°C	30°C	40°C	50°C	60°C
S.L.	1.0	1.0	1.0	1.0	1.0
25	1.0	1.0	1.0	1.0	1.0
30	1.0	1.0	1.0	1.0	0.5
40	1.0	1.0	0.25	0.0078	0.0625
50	1.0	0.125	0.0078	0.0005	------
60	0.06	0.0005	------	------	------

It is recommended that the motor life be computed for sea level conditions and 22°C using equations of Table 17. Life is then derated per Table 16. Alternately, the bearing temperature and winding temperature can be estimated from Table 8 and applied to the equations of Table 17.

3.4 FHP Motor Failure Model

In Sections 3.2 and 3.3 equations were developed to predict a bearing failure model and a winding failure model respectively. These two failure modes dominate the overall life of FHP motors. Since either failure may occur during the life of a motor they are considered as competing failure modes. In Reference 1 a competing failure model was developed based on the following distribution:

$$F(t) = 1 - e^{-\left[\left(\frac{t}{\alpha_B}\right)^{\beta_B} + \left(\frac{t}{\alpha_w}\right)^{\beta_w}\right]} \qquad (45)$$

where: $F(t)$ = failure percentile

t = time to failure (hrs)

α_B = bearing characteristic life (hrs)

β_B = Weibull slope for bearing failures

α_w = winding characteristic life (hrs)

β_w = Weibull slope for winding failures

By taking the natural logarithm, Equation (45) may be rewritten:

$$\ln(1-F(t)) = -\left[\left(\frac{t}{\alpha_B}\right)^{\beta_B} + \left(\frac{t}{\alpha_w}\right)^{\beta_w}\right]$$

To solve this equation for motor life (t) knowledge of the Weibull slope and characteristic life must be obtained. Equations (26) and (27) describe the failure model for bearings providing the value of α_B. Equation (26) requires considerable knowledge regarding the motor and bearing while Equation (27) necessitates knowledge of the ambient temperature only at the sacrifice of approximately 11 percent in the accuracy of the prediction. Figure 5 is a distribution of Weibull slopes indicating a mean slope of β_B of 2.878. It will be recognized immediately that solution of Equation (45) will require iteration techniques since t/α_b is raised to the 2.878 power. Changing the equation to a cubic, which is within one standard deviation, simplifies the solution of the equation. In the general form therefore, a value of $\beta_B = 3$ is suggested while $\beta_B = 2.878$ is suggested for the more complex solution.

Winding characteristic life α_w can be computed from Equations (40), (41), or (42). Equation (40) is the simplified general winding life model using ambient temperature. Equations (41) or (42) are the complex form including motor size, quality and winding temperature. The Weibull slope for winding failures as outlined in Table 15 is $\beta_w = 1.0$ for all motors except those 1 inch (2.54 cm) outer diameter of the stator, operating at winding temperatures of 160°C or greater. For this special case $\beta_w = 2.329$.

Table 17 summarizes the overall FHP motor failure models from Equation (45),

TABLE 17

FHP Motor Life Prediction

A. General Case (simplified Model)

A-1 $$\ln(1-F(t)) = -\left[\left(\frac{t}{\alpha_B}\right)^3 + \frac{t}{\alpha_w}\right]$$

where:

A-2 $$\frac{1}{\alpha_B} = \frac{1}{10^{\left[\frac{2357}{T_{amb}} - 2.534\right]}} + \frac{1}{10^{\left[-\frac{4500}{T_{amb}} + 20\right]} + 300}$$

A-3 $$\text{Log } \alpha_w = \frac{2357}{T_{amb}} - 1.83$$

(continued)

TABLE 17 (continued from previous page)

B. Complex Case (complete model)

(Equation B-1): $\ln\left[1-F(t)\right] = -\left[\left(\frac{t}{\alpha_B}\right)^{2.878} + \frac{t}{\alpha_w}\right]$ for motors greater than 1 in (2.54 cm) stator outer diameter and all motors with winding temperature less than 159°C.

(Equation B-2): $\ln\left[1-F(t)\right] = -\left[\left(\frac{t}{\alpha_b}\right)^{2.878} + \left(\frac{t}{\alpha_w}\right)^{2.329}\right]$ for motors equal or less than 1 in. (2.54 cm) stator outer diameter at winding temperature of 160°C or greater

where:

(Equation B-3): $$\frac{1}{\alpha_B} = \frac{1}{10^{\left[\frac{2342}{T_B} + q - 4.32\ DN \times 10^{-6} - K_g - .001N\left(\frac{W}{SP}\right)^{1.5}\right]}} + \frac{1}{10^{\left[\frac{-4760}{T_B} + 19.7\right]} + 300}$$

(Equation B-4): $\text{Log}\ \alpha_w = \frac{2342}{T_w} - 1.206 + q_w$ (for use with Equation B-1)

(Equation B-5): $$\frac{1}{\alpha_w} = \frac{1}{10^{\left[\frac{2342}{T_w} - 1.206 + q_w\right]}} + \frac{1}{10^{\left[\frac{4500}{T_w} - 6.328 - q_w\right]}}$$

(for use with Equation B-2)

It is obvious from inspection of Table 17 that the simplified model offers considerable advantage in the ease of use. Table 6 provided an estimate of the accuracy of prediction using the complex model. It is noted that 75 percent of the failures would be predicted within a life model. Although the prediction accuracy is improved 11 percent, the ability to estimate all constants in the complex model decreases the probable prediction accuracy. The simplified model therefore, was selected as the general prediction model for use in the reliability handbook with stipulations regarding its suitability for extremes of temperature, speed, load and motor size. For those type applications the more complex model is recommended.

3.4.1 Failure Rates of FHP Motors

The method most often used to analyze failure populations is thru determination of the failure rate of the population. The failure rate also termed the hazard rate over the interval $t_i < t \leq t_1 + \Delta t_1$ is defined as the ratio of the number of failures occurring in the time interval to the number of survivors at the beginning of the time interval divided by the length of the time interval; i.e.:

$$h(t) = \frac{\left[n(t_i) - n(t_1 + \Delta t_1\right]/n(t_1)\ 100}{\Delta t_i} \quad \%\ \text{per hour}$$

The hazard rate is a measure of the instantaneous rate of failure and is useful in describing the type of failure. Increasing failure rates signify a wear out process; constant failure rates indicate a random failure mode and decreasing failure rates are associated with infant mortality failures generally due to design weakness or assembly defects.

For increasing failure rates such as encountered with bearings, the instantaneous failure rate at any time (t) is not a good estimate of the number of failures that have occurred to time (t). The average failure rate $\bar{H}$ (t) of the motor at the 50 percentile failure is the suggested failure rate to be combined with the other constant failure rate components. The average failure rate can be defined as:

$$\bar{H}(t)_{avg} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} h(t)\, dt \qquad (46)$$

If the average failure rate to any time (t_2) from time 0 is utilized, Equation (46) reduces to

$$\bar{H}(t)_{avg} = \frac{1}{t_2} \int_0^{t_2} h(t)\, dt$$

However, the integral of h(t) is the cumulative hazard function H(t). Therefore

$$\bar{H}(t)_{avg} = \frac{1}{t} H(t)$$

For the competing failure model, the cumulative hazard function was defined as:

$$H(t) = \left(\frac{t}{\alpha_B}\right)^{\beta_B} + \left(\frac{t}{\alpha_W}\right)^{\beta_W} \qquad (47)$$

The average failure rate, therefore, is

$$\bar{H}(t)_{avg} = \frac{H(t)}{t}$$

$$\bar{H}(t)_{avg} = \frac{t^{\beta_B - 1}}{\alpha_B^{\beta_B}} + \frac{t^{\beta_W - 1}}{\alpha_W^{\beta_W}} \qquad (48)$$

Table 17 defines the values of α_B, α_W, β_B, and β_W. The time (t) is the motor life as calculated from the equations of Table 17. If we consider only the simple case of Table 17, Equation (48) can be rewritten as:

$$\overline{H}(t)_{avg} = \left[\frac{t^2}{\alpha_B{}^3} + \frac{1}{\alpha_w} \times\right] 10^6 \text{ failures}/10^6 \text{ hours} \quad (48a)$$

where
$$\frac{1}{\alpha_B} = \frac{1}{10^{\left(\frac{2357}{T_{amb}} - 2.534\right)}} + \frac{1}{10^{\left(\frac{-4500}{T_{amb}}\right) + 20} + 300}$$

$$\alpha_w = \frac{2357}{T_{amb}} - 1.83$$

and t is calculated from:

$$\ln 1\text{-}F(t) = -\left[\frac{t}{\alpha_B}^3 + \frac{t}{\alpha_w}\right] \quad (49)$$

The mean time to failure, as previously discussed, is estimated at the 50 percentile failure condition where F(t) = 0.5. The natural log of 0.5 is -.69315. Equation (49) may be rewritten as:

$$.69315 = \frac{t}{\alpha_B}^3 + \frac{t}{\alpha_w}$$

This is a cubic in t and may be solved for t. (50)

$$t = \alpha_B \left[\sqrt[3]{.34657 + \sqrt{.12011 + .03704 \left(\frac{\alpha_\beta}{\alpha_w}\right)^3}} + \sqrt[3]{.34657 - \sqrt{.12011 + .03704 \left(\frac{\alpha_\beta}{\alpha_w}\right)^3}}\right]$$

3.4.2 Use of FHP Motor Failure Models

In order to assist the reader in the use of FHP motor failure models and prediction techniques, an example has been selected that utilizes the developed equations. Both the simplified model and the complex model are applied to illustrate predictions using both procedures.

It is desirable to obtain motor life and failure rate of a 1/4 HP 60 Hz motor that operates 50 percent of the time at an 18°C ambient and the remainder at 100°C ambient. The motor is assumed to be of commercial quality, standard end bell construction, a capacitor start induction motor with silicone lubricated bearings that are loaded with a ten pound load per bearing.

3.4.2.1 Simplified Model

In this model, it is necessary to use only the ambient temperature for prediction of motor life and failure rate. In the case cited, operation occurs at two different temperatures and bearing and winding life must be computed for both. From Table 17 Equation A-2 and A-3 are used.

a) Bearing Life (Eq. A-2)

$$\text{A-2} \quad \frac{1}{\alpha_{B_1}} = \frac{1}{10^{\left[\frac{2357}{T_{amb}} - 2.534\right]}} + \frac{1}{10^{\left[-\frac{4500}{T_{amb}} + 20\right]} + 300}$$

at 18°C Ambient

$$\frac{1}{\alpha_{B_1}} = \frac{1}{10^{\left[\frac{2357}{291} - 2.534\right]}} + \frac{1}{10^{\left[\frac{-4500}{291} + 20\right]} + 300}$$

$$\frac{1}{\alpha_{B_1}} = .00000272 + .0000288$$

$$\alpha_{B_1} = 31677 \text{ hrs}$$

at 100°C Ambient

$$\frac{1}{\alpha_{B_2}} = \frac{1}{10^{\left[\frac{2357}{373} - 2.534\right]}} + \frac{1}{10^{\left[\frac{-4500}{373} + 20\right]} + 300}$$

$$\frac{1}{\alpha_{B_2}} = .000164 + .000000011$$

$$\alpha_{B_2} = 6095 \text{ hrs}$$

Overall bearing life

Equation (21) is used to obtain life under thermal cycling conditions.

$$N\left(\frac{h_1}{\alpha_{B_1}} + \frac{h_2}{\alpha_{B_2}}\right) = 1$$

where N = no. of cycles, h_i = hrs. at T_i, and α_{B_i} = life at T_i.

By substituting $\frac{\alpha_B}{h_1 + h_2}$ for N where α_B = cycling characteristic life,

then:
$$\frac{h_1}{h_1 + h_2}\left(\frac{\alpha_B}{\alpha_{B_1}}\right) + \frac{h_2}{h_1 + h_2}\left(\frac{\alpha_B}{\alpha_{B_2}}\right) = 1$$

Since proportion of time at each temperature = .5(or 50%), we have:

$$\frac{.5\ \alpha_B}{\alpha_{B_1}} + \frac{.5\ \alpha_B}{\alpha_{B_2}} = 1$$

or
$$\frac{.5\ \alpha_B}{31677} + \frac{.5\ \alpha_B}{6095} = 1$$

$$\alpha_B = 10223 \text{ hr.}$$

b) Winding Life (Eq. A-3)

at 18° Ambient

$$\text{Log } \alpha_{w_1} = \frac{2357}{18 + 273} - 1.83$$

$$\alpha_{w_1} = 1860614 \text{ hrs}$$

at 100°C Ambient

$$\text{Log } \alpha_{w_2} = \frac{2357}{373} - 1.83$$

$$\alpha_{w_2} = 30834 \text{ hrs}$$

Overall Life (Equation 21)

$$\frac{.5\ \alpha_w}{1860614} + \frac{.5\ \alpha_w}{30834} = 1$$

$$\alpha_w = 60663 \text{ hrs}$$

c) Motor Life (Equation A-1, Equation 49)

$$\cdot\ \ln(1-F(t)) = -\left[\left(\frac{t}{\alpha_B}\right)^3 + \frac{t}{\alpha_w}\right] \qquad \text{(A-1)}$$

at F(t) = 0.5 use Equation (50)

$$t = 10223\left[\sqrt[3]{.693394} + \sqrt[3]{-.000255}\right]$$

$$t = 10223\left[.8851 - .06337\right]$$

$$t = 8400 \text{ hrs}$$

d) Motor failure rate (Equation 48A)

$$\lambda_t = \left[\frac{t^2}{\alpha_B^{\ 3}} + \frac{t}{\alpha_w}\right] \times 10^6$$

$$\lambda_t = \left[\frac{8400^2}{10223^3} + \frac{1}{60663}\right] \times 10^6$$

$$\lambda_t = 82.5 \text{ failures}/10^6 \text{ hrs}$$

3.4.2.2 Complex Model

In this model it is necessary to obtain winding temperature, bearing temperature and bearing characteristics. To obtain the operating temperature refer to Table 8. For the motor in question, the high slip category is used with temperature rises selected at low slip; i.e. motor temperature rise 30°C, bearing temperature rise 15°C. For more accurate results the manufacturer should be consulted.

a) Bearing Life (Table 17 Equation B-3)

It is necessary to establish quality, grease type, DN and load as follows:

quality (q): See Equation (17); commercial q = -.27

grease (K_g): See Section 3.2.2.6 grease 5; K_g = 1.81

DN : See Table 9

.25HP = 186 watts; D = 13 mm

Equation (20): $N = \frac{120 \times 60}{2} = 3600$ RPM

DN = 46800

W : given as 10 lbs

SP for 13 mm bearing: (Specific load at 33-1/3 rpm) See brg. mfgr. catalog for 0.5 in. brg.

SP = 505 (R-8 brg)

T_B : $18 + 15 + 273 = 306^{o}K$

and

$100 + 15 + 273 = 388^{o}C$

Brg. Life at 18°C

From Equation (18)

$$\text{Log life}_{\beta} = \frac{2342}{T_B} + q - 4.32\ DN \times 10^{-6} - K_g - .001\ N\left[\frac{W}{SP}\right]^{1.5}$$

$$= \frac{2342}{306} - .27 - 4.32(.0468) - 1.81 - 3.6\left[\frac{10}{505}\right]^{1.5}$$

$$= .00000435 + .000070$$

$$\alpha_{B_1} = 13414 \text{ hrs}$$

Brg. Life at 100°C

Equation (18) partially solved above; i.e.

$$\text{Log life} = \frac{2342}{388} - 2.293$$

Log life = 3.743

Equation (B-3) as above

$$\frac{1}{\alpha_{B_2}} = \frac{1}{10^{3.743}} + \frac{1}{10^{\left[\frac{-4760}{388} + 19.7\right]} + 300}$$

$$\alpha_{B_2} = 5539 \text{ hrs}$$

Overall Bearing Life (Equation 21)

$$\frac{.5\ \alpha_B}{\alpha_{B_1}} + \frac{.5\ \alpha_B}{\alpha_{B_2}} = 1$$

$$\frac{.5\ \alpha_B}{13414} + \frac{.5\ \alpha_B}{5539} = 1$$

$$\alpha_B = 7840 \text{ hrs}$$

b) Winding Life (Table 17 Equation B-4)

$$\text{Log } \alpha_w = \frac{2342}{T_{wind}} - 1.206 + q_w$$

$$T_{w_1} = 18 + 30 + 273 = 321^{\circ}\text{K}$$

$$T_{w_2} = 100 + 30 + 273 = 403^{\circ}\text{K}$$

From Equation (42) $q_w = .13$

Life at 18°C

$$\text{Log } \alpha_{w_1} = \frac{2342}{321} - 1.206 - .13$$

$$\alpha_{w_1} = 311906 \text{ hrs}$$

Life at 100°C

$$\text{Log } \alpha_{w_2} = \frac{2342}{403} - 1.206 - .13$$

$$\alpha_{w_2} = 29882 \text{ hrs}$$

Overall Winding Life (Equation 21)

$$\frac{.5\ \alpha_w}{911906} + \frac{.5\ \alpha_w}{29882} = 1$$

$$\alpha_w = 57868 \text{ hrs}$$

c) Motor Life (B_{50} Life) Table 17 Equation (B-1)

$$\ln(1-F(t)) = -\left[\frac{t}{\alpha_B}^{2.878} + \frac{t}{\alpha_w}\right]$$

$$\ln(1-0.5) = -0.693147$$

$$.693147 = \frac{t}{7840}^{2.878} + \frac{t}{57868}$$

This equation was solved by computer

$t = 6492$ hrs

d) Motor Failure Rate

$$\lambda_t = \left[\frac{t^{1.878}}{\alpha_B{}^{2.878}} + \frac{1}{\alpha_w}\right] \times 10^6 \text{ failures}/10^6 \text{ hrs}$$

$$\lambda_t = \left[\frac{6492^{1.878}}{7840^{2.878}} + \frac{1}{57868}\right] \times 10^6$$

$$\lambda_t = 107 \text{ failures}/10^6 \text{ hrs}$$

The results of the complex model indicate a failure rate λ_t of 107 while the simple model λ_t is 82.5.

4.0 CONCLUSIONS

A review of failures and failure modes of FHP motors was completed and a reliability prediction model developed. The following specific conslusions and recommendations were developed as a result of these studies:

1. Two failure modes dominate the life of FHP motors utilizing grease packed non-relubricated ball bearings; i.e. bearing failures and winding failures. Bearing failures represented approximately 80 percent of the total failures of these type motors. Winding failures were more prominent in the early life of the motor and were attributed to shorts and grounds resulting from assembly quality rather than long term insulation degradation. The winding failure percentage of small motors of 2.54 cm lamination OD operating at ambient temperatures of 125^{o}C increased to 50 percent which was attributed to the small wire size used in these motors.

2. Both failure modes were analyzed utilizing a Weibull cumulative distribution function. A Weibull shape parameter in the range of 2.8 to 3.0 was used to describe bearing failures indicating an increasing failure rate and wear out condition. Winding failures were described by a Weibull shape parameter of 1.0 indicative of a constant failure rate and random type failure.

3. Multi-variable linear regression techniques were utilized to develop a mathematical model of both failure modes. Life of the bearings and windings was most sensitive to temperature of operation. A simplified model was developed for both failure modes as a function of ambient temperature only. This model proved sufficiently accurate for general predictions of motor life. Secondary influences including motor load, speed, grease type, winding temperature use, bearing temperature rise and quality were used to develop a more complex model for life prediction. This model was suggested when unusual operating con-

ditions were anticipated. The added complexity of this model was not deemed necessary for most lightly loaded FHP motor applications.

4. The two failure modes were recombined as a competing risk failure model for predicting overall motor life. Equations were also developed for prediction of the average failure rate using the 50 percentile failure point as an estimate of the mean time to failure.

5. Appendix B was developed as a recommended "failure prediction model" replacement for both fractional horsepower motors and electronic cooling devices in MIL-HDK-217B.

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6.0 LIST OF SYMBOLS

A	Constant
B	Constant
B_b	Denotes bearing failure
^oC	Temperature, degrees centigrade
C.	Characteristic
C_c	Denotes combined bearing and winding failures
C_k	Total squared error coefficient
D	Bearing bore diameter in millimeters
D.F.	Degrees of freedom
DN	Bore diameter (mm) times speed (rpm)
DN_L	Limiting DN
e	Constant
E	Constant
f	Line frequency hz
F	Statistical, F-test value
f(t)	Statistical, probability density function
F(t)	Statistical, cumulative failure distribution function
G	Dimensionless material parameters
h	Time at temperature (hours)
hrs.	Hours
h(t)	Statistical, hazard function
H	Dimensionless film thickness
H(t)	Statistical, cumulative hazard function
$\overline{H}(t)_{avg}$	Statistical, average failure rate
Hz	Cycles per second
^oK	Degrees Kelvin
k	Number of parameters in regression
K_g	Grease constant
L	Life (unless noted otherwise, L is the Weibull characteristic life when F(t) = .632)
Log	Log to base 10 (common logarithms)
LQG	Lower quality grade
ln	Log to base e (2.71828) (natural logarithms)

M	Multiplier
MSE_k	Mean sum of squares of errors
n	Number of units or thermal cycles
N	Speed in revolutions per minute, or number of data points
ORIG	Original
P	Number of poles
q	Quality code
q_w	Quality code for winding failures
R	Correlation coefficient
R^2	Square of the correlation coefficient
R^2_k	Coefficient of multiple correlation
RPM	Revolutions per minute
S.D.	Standard deviation
S.P.	Specific load
SQS	Squares
SS	Sum of squares
t	Time in hours
T	Temperature, ^oK
T_t	Life correction factor
U	Dimensionless speed
UQG	Upper quality grade
W	Load in pounds
W_e	Denotes winding failures
X	Independent variable
$\overline{X}$	Average of all independent data values
X_i	i^{th} independent variable data value
$\hat{X}_i$	Calculated data value
Y	Dependent variable
Y	Average of all dependent variable data values
Y_i	i^{th} dependent variable data value
$\hat{Y}_i$	Calculated Data value
α	Weibull characteristic life
β	Weibull shape parameter
λ	residual error

μ_o	Lubricant viscosity
σ	Standard deviation
%	Percent

SUBSCRIPTS

amb	refers to ambient
avg	refers to average
B	refers to bearings
i	refers to ith data value
w	refers to winding
x	refers to independent variables
y	refers to dependent variables

APPENDIX A

MULTIPLE LINEAR REGRESSION ANALYSIS TECHNIQUES & EXAMPLES

A. INTRODUCTION

This appendix contains a general description of applied regression procedures. Application of these procedures is widely used today by management and scientific personnel. Successful use requires an understanding of the underlying theory as well as the practical data problems which can arise from everyday applications.

The following subjects are addressed:

Regression Analysis
Use of Regression Analysis
Correlation Coefficients and Matrix Arrays
Correlation Coefficients of Transformed Data
Selecting an "Optimum Set" of Explaining Parameters
A Large Number of Independent Variables
Regression Tables Explained
Examples of Data Reduction

A.1 Regression Analysis

Regression Analysis is a technique which can be used to describe and establish relationships between two or more sets of data. It is particularly useful in cases where the data is statistical in nature and has scatter if plotted graphically. The relationships derived from this technique may not always provide a perfect description of associated data; nevertheless, it can be a powerful tool even when approximations to data variations are found. Problems that could only be handled by guesswork can be confidently handled once interrelationships are established with regression techniques.

A.2 Use of Regression Analysis

Regression procedures can be used to:

a) describe

b) control, or

c) predict

In each case one must use a set of data which is numeric in form. In some instances the data can be separated into either dependent or independent groups. The dependent response variables are usually what is to be described, controlled, or predicted. In other cases, a knowledge of which variables are "independent" are not known before examining the data. Regression procedures can be employed to establish the interdependence of such sets of data.

In some fields, engineering concepts or models can be used to help select the independent variables to be employed in the regression relationship. Once established, the functional form of the relationship can be used to predict or control the system response from a given set of independent variable values.

The power of regression analysis is best used whenever the response of the system to be described is affected by several operating parameters simultaneously. Often times system response to a single independent operation parameter is not available. Either single variable control tests on the systems are much too costly or cannot be implemented in practice. Given a collection of data, regression procedures can provide insight as to which variables, if any, are most useful in describing the system response.

A.3 Correlation Coefficients

The association between two variables is often examined by plotting in graphic form one variable against the other. Whenever a large number of variables are involved this process is not always practical. The calculation of a correlation coefficient is one screening tecnnique which avoids data plotting but can still provide information on data interdependence. The "correlation coefficient" defined by (R) below is one measure of the degree of association between two variables.

$$R = \frac{\frac{1}{N}\sum_{i=1}^{N} (X_i - \overline{X})\ (Y_i - \overline{Y})}{\sigma_x\ \sigma_y}$$

The values represented by $\overline{X}$ and $\overline{Y}$ are the means of all the X_i and Y_i values whose association is being tested. The σ_x, σ_y are the standard deviations of all X_i and Y_i values. A positive or negative (R) results whenever the slope of the association between variables is positive or negative, respectively.

It can be shown that the correlation coefficient (R) is always within the range:

$$-1 \leq R \leq 1$$

A sample set of correlation coefficients is shown in Figure A-1 with a display of the graphic data used to obtain the coefficients. As shown, the closer the (R) value is to the extreme values of (+1) or (-1) the less scatter there is in the data. As (R) approaches zero the scatter approaches a random pattern for the variables being tested.

A.4 Correlation Matrix

Correlation coefficients are often calculated for more than just one pair of variables. In fact for any number of (k) variables there can be

$$\frac{k(k-2)}{2}$$

unique pairs of linear associations which might be evaluated. Correlation coefficients are often displayed in array or matrix form for convenience. The elements of the array are the (R) values calculated from each pair of variables whose association is being tested. The row and column numbers of the matrix merely represent the variables for which the correlation coefficients have been calculated. For example, see Figure A-2. The last column of that figure is all (R) values for the interdependence between temperature and the characteristic life of the FHP motor. The row and column variable assignments are as tabulated at the bottom of the figure. The first row-column entry (.99) is a number very close to (1.00) and represents the high correlation of the ambient temperature data with itself. All diagonal elements of a correlation matrix will be very near 1.00 if a linear association between variables is being tested. In Figure A-2, the diagonal elements are all one or near one, but are shown as (-.##) due to an overflow in the format print condition of the computer printout.

The correlation matrix is a convenient way to display all tabulated associations, (R) values, whenever two or more sets of data variables are being evaluated.

A.5 Correlation Coefficients of Transformed Data

The correlation coefficient (or matrix array) is a numeric procedure whereby the potential relationship between two sets of data can be examined. Nonlinear as well as linear relationships can be tested for their relative degree of association if the data are first transformed. Data interdependence which can be linearized on log-log or semi-log plots can be inspected analytically by taking the appropriate logarithms of the data before computing the correlation coefficients. Systems governed by the laws of physics can often be explained by one of these functional relations. Testing for one of these relations at the onset of examining the data can provide quick review of the most obvious data dependencies.

A.6 Selecting an "Optimum Set" of Explaining Parameters

The "best" regression equation is the one which can sufficiently describe the system under study with the smallest number of variable parameters. There is no unique statistical procedure that will provide the "best" descriptive equation in any given instance. Personal judgment is always required. Scientific insight and engineering principles should never be substituted by some automatic statistical screening method for obtaining the "best" model of representation of a given set of data. Some analytical procedures are more useful than others in helping find an "optimum set" of parameters that can sufficiently describe the system or data being studied. Two types of tests are common; parameter acceptability tests and optimization criteria for gathering the "best" set of parameters.

Single or multiple parameter acceptability can be established for a linear set of data through the use of the F-test

$$F = \frac{\text{Mean Square of the Regression Components}}{\text{Mean Square of the Error Components}}$$

Where a regression and error component are defined as follows:

Regression components = $(\hat{Y}_i - \bar{Y})$

Error components = $(Y_i - \hat{Y}_i)$

Each is shown graphically for a single parameter fit in Figure A-3. This ratio can be calculated sequentially as parameters are being added to the model or collectively to all parameters in the model.

The (F) number for parameter acceptability can be established by using any of a number of prepared tables which show the level of significance for any number of data points and parameters in the model. In general, the higher the F-number the better chance there will be of the model successfully describing some additional set of data similar to that from which the model was developed.

Other analytical means are often used to establish an "optimum set" of parameters from those available. The methods commonly used involve the determination of all errors associated with the data and the explaining model. The errors being defined as the differences between the actual data and the corresponding predicted ones.

Three calculating procedures or criteria are helpful in determining whether one combination of parameters is better than another. Each is calculated as follows:

$$R_k^2 = 1 - \frac{\text{Sum of Squares of the Errors}}{\text{Total Sum of Squares}} \quad \frac{\sum(Y_i - \hat{Y}_1)^2}{\sum(Y_i - \bar{Y})^2}$$

$$MSE_k = \frac{\text{Sum of Squares of the Errors} \quad \sum(Y_i - \hat{Y}_i)^2}{(N-k-1)}$$

$$C_k = \frac{(\text{Sum of Squares of the Errors})}{\text{Mean sum of Squares of Errors for all Possible Variables in Regression}} - (N - 2\ (k+1))$$

where N is the number of data points and k is the number of parameters entered into the regression model. Note that when no parameters are in regression the sum of the squares of the errors is taken as the total sum of the squares.

The following comments are pertinent to the use of the three criteria:

Criterion	Comments	Value Approached as Number of Variables in Model Increases
R_k^2	Does not account for the number of variables in model.	Rapidly approaches 1
	Change in R_k^2 often very small as number of variables in model increases	
MSE_k	May increase slightly upon adding model parameters	Approaches a minimum (the variance of the errors)
	Changes in MSE_k often very small as number of variables increase	
C_k	Optimum test for finding models that give the smallest squared error	Approaches k + 1 for "best" set of variables. (k = number of parameters in model.)
	Minimum C_k occurs for models containing either bias or random errors.	

The computed values of R_k^2, MSE_k, and C_k are shown graphically in Figure A-4. In each case those combinations of variables providing a value of R_k^2, MSE_k, or C_k that lies nearest the solid line (the left boundary of the shaded regions shown) will likely be the optimum combination of variables for modelling. The final selection, of course, being made from a scientific knowledge of the system being studied.

A.7 A Large Number of Independent Variables

The "optimum" selection process cannot always be used when a large number of independent parameters exist. Since each potential independent parameter can be included or excluded from the model there are (2^{k-1}) possible linear regression equations that exist with k possible parameters. In addition, a few parameters may become a large collection of different parameters if cross products, logarithms, or other typical ways of modifying the data are to be examined. Even with the availability of large computers running all possible regression models for review becomes unwieldy. In the case of a large number of available parameters other procedures must be employed.

Among those used are the:

a) Forward selection technique,
b) Backward elimination procedure, and
c) Stepwise procedure.

The objective of each procedure is to give the best description of the response data (dependent variable) with the fewest number of convenient parameters in the functional expression. Unfortunately the above procedures do not always lead to the same solution for a given set of data. In those instances where the data variables are not highly interrelated the above procedures will often achieve the same solution. In cases of highly interrelated parameters a unique description of the data is not always available. In these instances the "best" solution will require a value judgment or minimum errors criteria test to make the final selection.

The forward selection technique inserts variables into the regression model until a satisfactory equation is achieved. The order of insertion is usually based on a calculation of the fractional correlation coefficient between the existing model and the parameters not yet entered. The parameter with the highest coefficient is entered next.

The backward elimination method begins with all variables in regression. Parameters are then removed sequentially. The procedure stops when the explaining equation is sufficient to describe the data. Partial correlation coefficients and F-values are used to establish which, if any, of the parameters should be taken out of the regression equation at each step.

The stepwise procedure is an improved version of the forward selection process. The improvement involves a re-examination of each parameter in regression after one is entered. A variable previously entered may be found to be unnecessary after other parameters have been brought into the model. The partial F-test is applied to each variable after each parameter is entered into regression. If any variable is shown to be nonsignificant at any stage, it is immediately removed from the model. The entry-removal procedure continues until the model is found to be acceptable.

A further improvement of this standard technique can be made by running the C_k test (see previous discussion) on those parameters that are in the model when the stepwise procedure is ended. In doing so, a fewer number of parameters may be found which yield an equivalent solution. In any case, the search method does not require that all regressions be analyzed.

A.8 Regression Tables Explained

Every useful computerized regression program has a tabularized output. Most programs provide a similar set of output results convenient for many different applications. Figure A-5 is a sample analytical output sheet with annotated explanation of the pertinent elements. More of these tables will be presented below without annotation.

A.9 Examples of Data Analysis Reduction

As shown previously in Figure 1, a knowledge of the motor temperature is very significant in estimating how long it will last. In fact, the inverse of the absolute motor temperature has a correlation coefficient of R=.86 (See Figure A-2 column 16). As shown in Figure A-2 this is the best transformation of the temperature data that was explored. All other temperature transformations showed a lower degree of correlation with motor life.

The practical aspects of getting available temperature data tends to make the information provided by the ambient operating temperature more useful than the others. When one wants to predict motor lives he does not always have the motor bearing or winding temperature at hand. The ambient motor operating temperature, however, is usually available. In order to get a feel for the relative importance of ambient and bearing temperature, a single variable "fit" to the life data was performed for each.

Tabular outputs of the regression information for the bearing related temperatures and the ambient operating temperatures are shown in Figures A-6 and A-7. The "best fit equations" from those figures are:

$$\text{Ambient Temp.} \quad \text{Log (C. Life)} = \frac{2357}{T_{amb}+273} - 2.534$$

$$\text{Bearing Temp.} \quad \text{Log (C. Life)} = \frac{2342}{T_{B}+273} - 1.908$$

Each of these variables are shown to be significant in estimating motor life. Both have F-values in excess of one hundred.

Bearing temperatures show a better "fit" to the data than the ambient (i.e. F-value of 159 versus 124). In addition, once the bearing temperatures are entered into regression, there is no justification for using the ambient or winding temperature information. This conclusion is drawn from the small F-values related to the ambient and winding temperatures shown in

Figure A-6 (T_{amb} F-value = .407 and T_{wind} F-value = .115 after T_{brg} had been accounted for in the regression equation If, however, the ambient temperature has been entered into regression, there is still significant ambient or winding temperature information left in the data as shown in Figure A-7 (T_{wnd} F-value = 10.0 and T_{brg} F-value = 18.7). Even though the bearing temperature is more significantly correlated with the life data as a practical engineering matter it was decided that the ambient temperature should be used in estimating motor lives. Since ambient motor operating temperatures are more readily available and are highly significant as shown, T_{amb} was chosen as one of the "primary predictors" of motor life.

Other parameters chosen as life predictors were reviewed extensively in the main body of this report. As explained in the previous discussion, the selection of an optimum set of "predictor" variables makes use of several regression procedures. The single interrelationship between the logarithm of motor life and listed variables can be seen in Figures A-8 and A-9. Column sixteen, the last column of each matrix shown, reveals typical single variable dependence (R values) with respect to motor life. Some comments are appropriate.

The first three coefficients in column sixteen of Figure A-8 again show the dependence of life on absolute temperature. Temperature shows the strongest correlation of any of the variables listed.

The fourth element of column sixteen, however, is misleading as shown. There is no apparent dependence (R=.00) of life on the D x N (bearing diameter times rotational speed of the shaft). The speed dependence does not show up until after the data has been corrected for the stronger effects such as temperature and type of grease used in the motor bearings.

The fact that motor life is dependent on D x N is shown quite well in Figures A-6 and A-7. Although D x N is listed as "not entered" into regression, the partial F-values are shown. These partial F-values represent the F-value that would be obtained if this parameter were to be included next in the explaining equations. The F-values of 3.8 (Figure A-6) and 5.4

(Figure A-7) represent a highly significant dependence between D x N and motor life. Other variables found to be significant after accounting for temperature were the quality code (listed as item 7 in Figures A-7 and A-9) and grease.

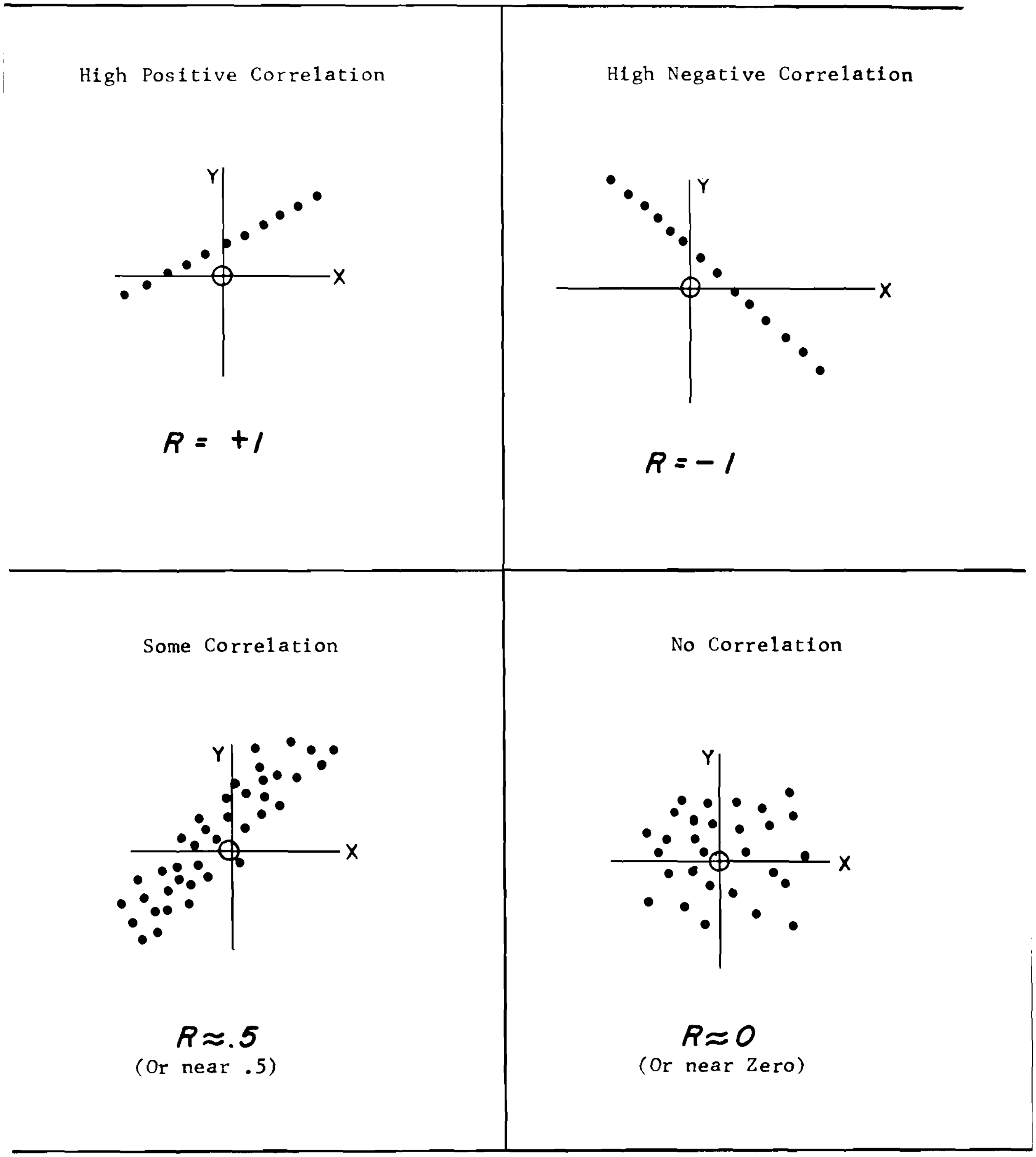


Figure A-1

Schematic Plots Depicting Various Amounts of Scatter, as Well as Correlation Coefficient (R) between Variable X and Response Y

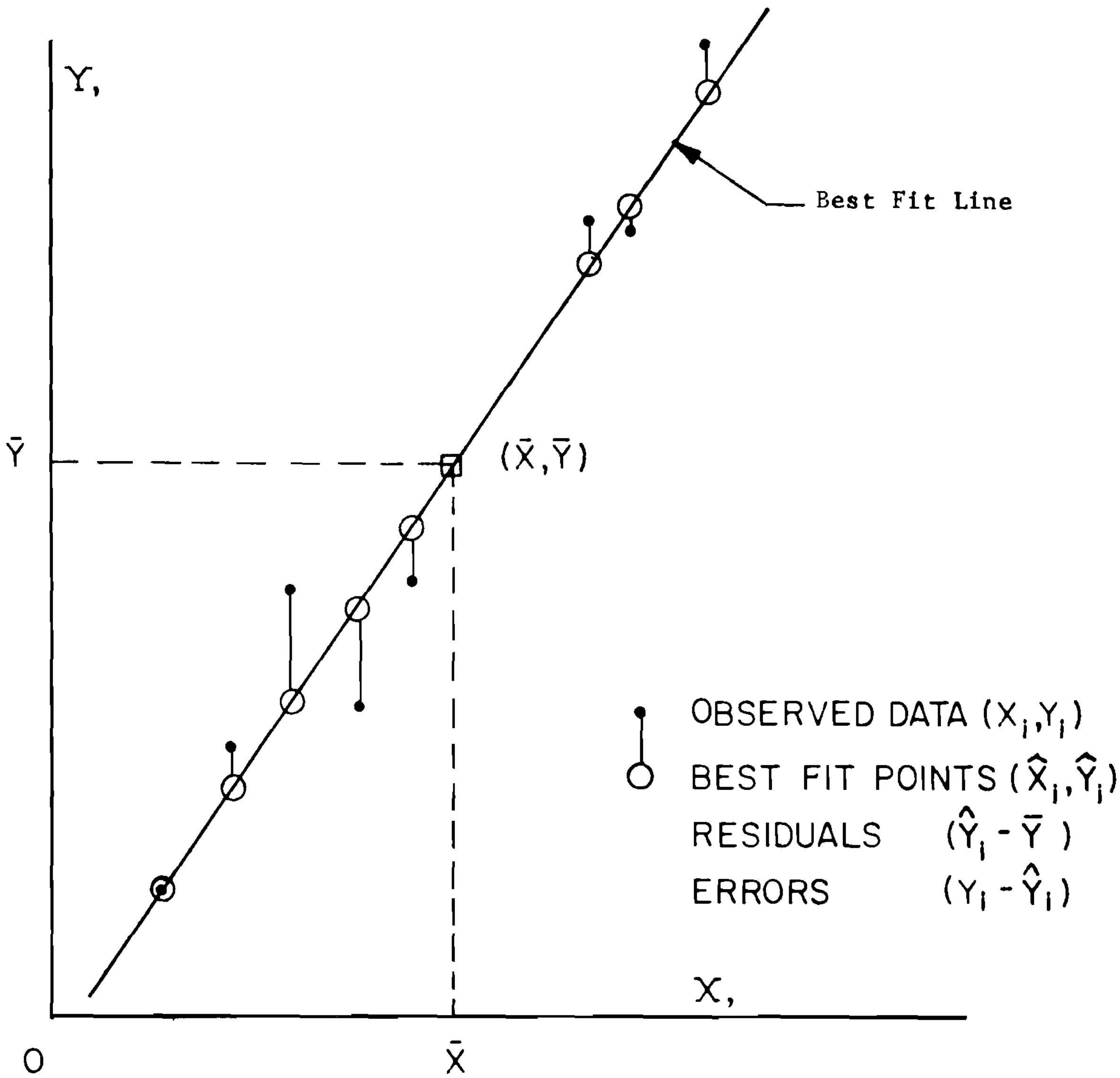
.99	.96	.96	-.63	-.59	-.57	-.08	-.10	-.08	.81	.75	.72	.72	.66	.64	-.17
.96	-.##	.99	-.57	-.58	-.56	-.02	-.08	-.05	.82	.79	.76	.74	.70	.68	-.13
.96	.99	-.##	-.56	-.56	-.54	.01	-.04	-.01	.84	.81	.79	.77	.72	.71	-.10
-.63	-.57	-.56	.99	.97	.96	.66	.68	.67	-.22	-.12	-.09	-.10	-.01	.01	.56
-.59	-.58	-.56	.97	.99	.99	.71	.75	.73	-.15	-.08	-.04	-.03	.04	.07	.60
-.57	-.56	-.54	.96	.99	.99	.74	.78	.77	-.11	-.03	-.00	.00	.08	.11	.66
-.08	-.02	.01	.66	.71	.74	.99	.98	.99	.49	.57	.60	.60	.67	.70	.86
-.10	-.08	-.04	.68	.75	.78	.98	.99	.99	.46	.53	.56	.57	.63	.66	.86
-.08	-.05	-.01	.67	.73	.77	.99	.99	.99	.48	.55	.58	.60	.66	.68	.87
.81	.82	.84	-.22	-.15	-.11	.49	.46	.48	-.##	.98	.98	.99	.97	.96	.36
.75	.79	.81	-.12	-.08	-.03	.57	.53	.55	.98	-.##	.99	.99	.99	.98	.43
.72	.76	.79	-.09	-.04	-.00	.60	.56	.58	.98	.99	-.##	.99	.99	.99	.45
.72	.74	.77	-.10	-.03	.00	.60	.57	.60	.99	.99	.99	-.##	.99	.98	.47
.66	.70	.72	-.01	.04	.08	.67	.63	.66	.97	.99	.99	.99	-.##	.99	.53
.64	.68	.71	.01	.07	.11	.70	.66	.68	.96	.98	.99	.98	.99	-.##	.54
-.17	-.13	-.10	.56	.60	.66	.86	.86	.87	.36	.43	.45	.47	.53	.54	-.##

TEMPERATURE CORRELATIONS WITH LOG(CHARACTERISTIC LIFE)

1 = AMBIENT
2 = BEARING
3 = WINDING
4 = 1/AMBIENT
5 = 1/BEARING
6 = 1/WINDING
7 = 1/(ABSOLUTE AMBIENT)
8 = 1/(ABSOLUTE BEARING)
9 = 1/(ABSOLUTE WINDING)
10 = LOG(AMBIENT)
11 = LOG(BEARING)
12 = LOG(WINDING)
13 = LOG(LOG(AMBIENT))
14 = LOG(LOG(BEARING))
15 = LOG(LOG(WINDING))
16 = LOG(CHARACTERISTIC LIFE)

Figure A-2

CORRELATION MATRIX FOR OVER 1000 FHP MOTOR FAILURES



N = Number of Data Points
i = i th Data Point

Figure A-3

Schematic Data Plot Showing Observed Data Points (X_i, Y_i), Best Fit Points (X_i, Y_i), Average $\bar{x} = \sum X_i/N$, $Y = \sum Y_i/N$ Error Values $(Y_i - Y_i)$, and Residuals $(Y_i - Y)$.

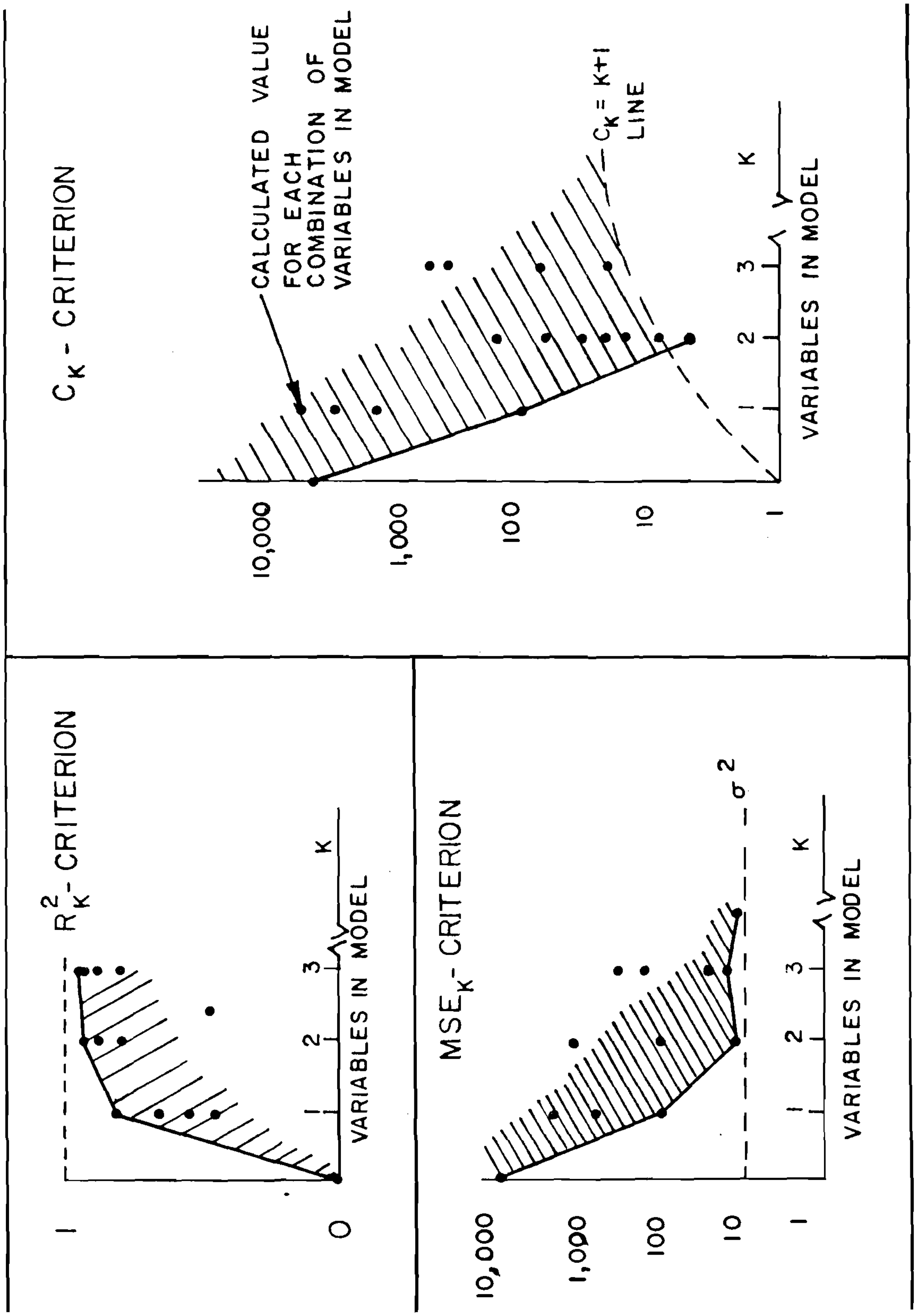


Figure A-4
Three Criteria for Testing the Relative "Best Fit" of a Combination of Variables in a Hypothetical Regression Model

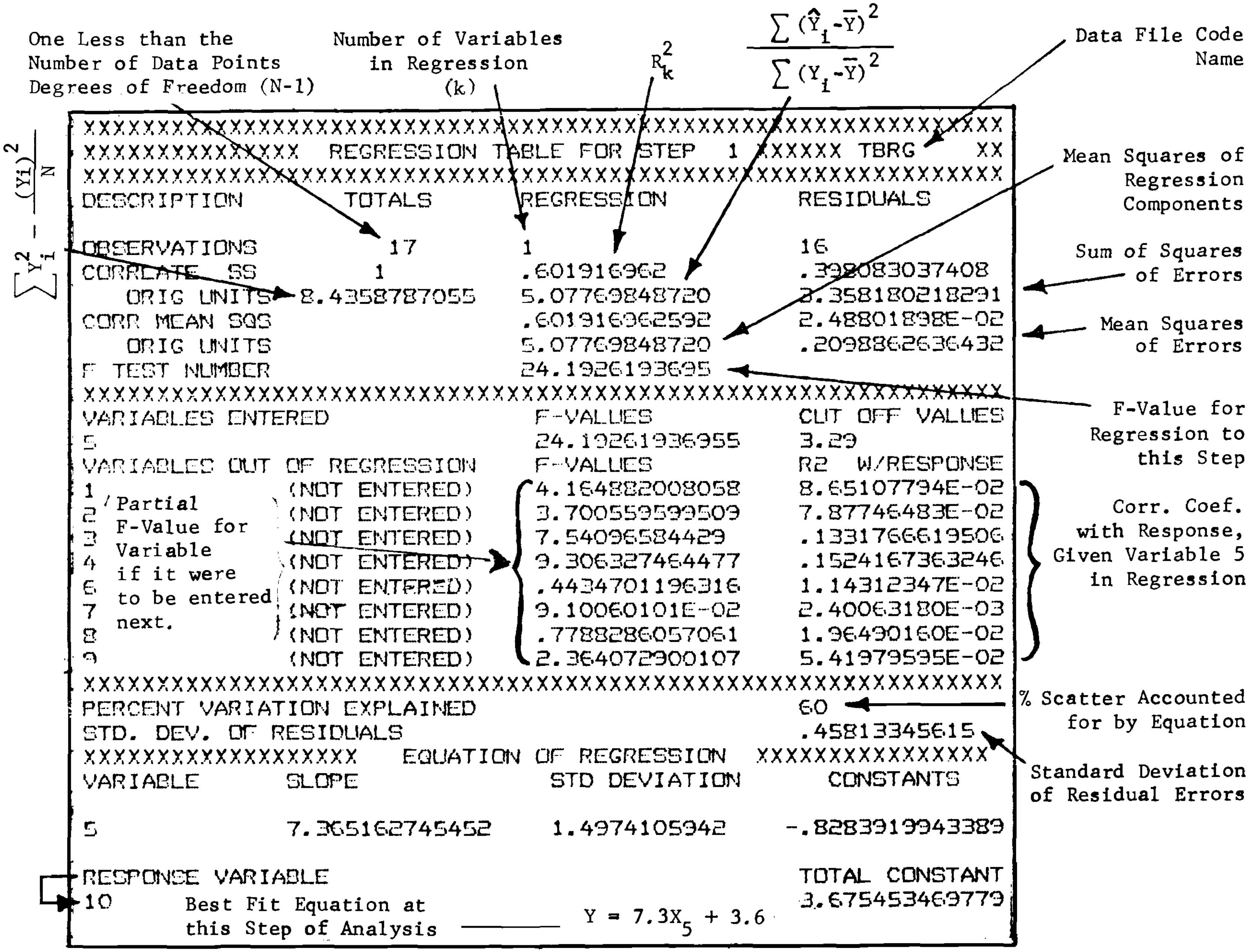


Figure A-5

Regression Table Description

```
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
XXXXXXXXXXXXXXX  REGRESSION TABLE FOR STEP   1 XXXXXX TERADC   XX
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
DESCRIPTION          TOTALS        REGRESSION            RESIDUALS

OBSERVATIONS            135        1                     134
CORRLATE  SS              1        .543523800251         .456476199749
   ORIG UNITS    53.883560694      29.28699767946        24.59656301454
CORR MEAN SQS                      .543523800251         3.40653880E-03
   ORIG UNITS                      29.28699767946        .183556440407
F TEST NUMBER                      159.5530923051
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
VARIABLES ENTERED                   F-VALUES             CUT OFF VALUES
T BRG ABS-1              NOTE:     159.5530923051        3.29
VARIABLES OUT OF REGRESSION         F-VALUES             R2  W/RESPONSE
T AMB ABS-1     (NOT ENTERED)      .4075622283483        1.39454206E-03
T WND ABS-1     (NOT ENTERED)      .1153236849238        3.95465495E-04
EARLY =1        (NOT ENTERED)      38.6195427391         .1027208313472
LATE   =1       (NOT ENTERED)      38.61954272783        .102720831324
U50 =1          (NOT ENTERED)      42.78808757997        .1111095972538
B35 =1          (NOT ENTERED)      .9609179303038        3.27435920E-03
D33 =1          (NOT ENTERED)      41.1387084556         .1078384091908
D X N           (NOT ENTERED)      3.82090228193         1.27476936E-02
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
PERCENT VARIATION EXPLAINED                              54
STD. DEV. OF RESIDUALS                                   .42843487301
XXXXXXXXXXXXXXXXXXX   EQUATION OF REGRESSION   XXXXXXXXXXXXXXXX
VARIABLE        SLOPE               STD DEVIATION        CONSTANTS

T BRG ABS-1     2342.767986407      185.47127948         5.550021179435

RESPONSE VARIABLE                                        TOTAL CONSTANT
LOG C.LIFE                                               -1.908089974872
```

Figure A-6

Regression Table for Bearing Temperature

```
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
XXXXXXXXXXXXXXX  REGRESSION TABLE FOR STEP  1 XXXXXX T6RADC  XX
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
DESCRIPTION          TOTALS      REGRESSION           RESIDUALS

OBSERVATIONS            135      1                    134
CORRLATE   SS          1         .480712282208        .519287717792
   ORIG UNITS  53.883560694      25.90248943471       27.98107125929
CORR MEAN SQS                    .480712282208        3.87528147E-03
   ORIG UNITS                    25.90248943471       .208813964621 6
F TEST NUMBER                    124.0457719465
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
VARIABLES ENTERED                 F-VALUES            CUT OFF VALUES
T AMB ABS-1             NOTE: --> 124.0457719466      3.29
VARIABLES OUT OF REGRESSION       F-VALUES            R2  W/RESPONSE
T WND ABS-1    (NOT ENTERED)      10.04881945347      3.64786549E-02
T BRG ABS-1    (NOT ENTERED)      18.76455763472      6.42060601E-02
EARLY =1       (NOT ENTERED)      39.0887788064       .1179526223525
LATE  =1       (NOT ENTERED)      39.08877878905      .117952622312
U50 =1         (NOT ENTERED)      51.99320038274      .1459482311328
B35 =1         (NOT ENTERED)      .2886183445159      1.12444680E-03
D33 =1         (NOT ENTERED)      54.23319985223      .1504147480364
D X N          (NOT ENTERED)      5.414027820727      2.03118007E-02
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
PERCENT VARIATION EXPLAINED                           48
STD. DEV. OF RESIDUALS                                .45696166647
XXXXXXXXXXXXXXXXXXX   EQUATION OF REGRESSION  XXXXXXXXXXXXXXXX
VARIABLE       SLOPE               STD DEVIATION        CONSTANTS

T AMB ABS-1    2357.544811901      211.67471          6.175985477438

RESPONSE VARIABLE                                     TOTAL CONSTANT
LOG C.LIFE                                           -2.534054272875
```

Figure A-7

Regression Table for Ambient Temperature

.99	.96	.97	.09	.08	.07	.46	.02	.68	.01	.13	.37	-.04	.75	.25	.73
.96	.99	.98	.13	.09	.09	.47	.03	.60	-.04	.16	.40	-.12	.73	.23	.77
.97	.98	-.##	.11	.09	.08	.45	.02	.64	-.04	.15	.40	-.08	.74	.20	.75
.09	.13	.11	-.##	-.15	-.12	-.03	-.01	.03	-.05	.31	.09	-.13	.06	.07	.00
.08	.09	.09	-.15	.99	.89	.28	-.06	.13	.33	-.25	-.04	-.09	.19	.22	.24
.07	.09	.08	-.12	.89	.99	.26	-.05	.11	.35	-.11	-.02	-.03	.19	.22	.17
.46	.47	.45	-.03	.28	.26	-.##	-.00	.20	.10	-.08	.05	-.48	.57	.46	.64
.02	.03	.02	-.01	-.06	-.05	-.00	-.##	-.00	-.04	-.06	.00	-.00	.06	.01	.02
.68	.60	.64	.03	.13	.11	.20	-.00	-.##	.26	.19	.25	.13	.77	.17	.39
.01	-.04	-.04	-.05	.33	.35	.10	-.04	.26	.99	-.15	-.07	.03	.10	.12	.00
.13	.16	.15	.31	-.25	-.11	-.08	-.06	.19	-.15	.99	.13	.09	.13	-.08	-.08
.37	.40	.40	.09	-.04	-.02	.05	.00	.25	-.07	.13	-.##	.09	.29	-.10	.18
-.04	-.12	-.08	-.13	-.09	-.03	-.48	-.00	.13	.03	.09	.09	-.##	-.10	-.22	-.44
.75	.73	.74	.06	.19	.19	.57	.06	.77	.10	.13	.29	-.10	-.##	.40	.63
.25	.23	.20	.07	.22	.22	.46	.01	.17	.12	-.08	-.10	-.22	.40	-.##	.44
.73	.77	.75	.00	.24	.17	.64	.02	.39	.00	-.08	.18	-.44	.63	.44	-.##

ROW AND COLUMN VARIABLES ASSIGNED AS FOLLOWS

1 = 1/(TEMP AMB ABSOLUTE)
2 = 1/(TEMP BRG ABSOLUTE)
3 = 1/(TEMP WNG ABSOLUTE)
4 = D*N (DIA. X RPM)
5 = NUMBER ON TEST
6 = NUMBER FAILED
7 = QUALITY CODE
8 = WEIBUL SLOPE
9 = TEST CORR COEF
10 = F-TEST VALUE
11 = WIRE SIZE
12 = NUMBER MOTOR POLES
13 = GREASE CODE
14 = TEST DATE
15 = SERIES
16 = LOG (CHARACTERISTIC LIFE)

CORRELATION MATRIX FOR OVER 1000 FHP MOTOR FAILURES

Figure A-8

.99	.99	.99	.41	.30	.16	.49	.05	.83	.07	.49	.48	.21	.92	.65	.76
.99	-.##	.99	.40	.30	.17	.50	.06	.84	.07	.49	.48	.21	.92	.68	.75
.99	.99	-.##	.40	.30	.17	.50	.06	.84	.07	.49	.48	.21	.92	.67	.75
.41	.40	.40	-.##	.11	-.08	-.15	-.04	.47	-.05	.72	-.00	.52	.42	-.16	.31
.30	.30	.30	.11	-.##	.46	.32	-.19	.33	.25	.32	-.01	-.03	.32	.24	.49
.16	.17	.17	-.08	.46	-.##	.32	-.09	.32	.46	-.00	.16	-.25	.32	.24	.11
.49	.50	.50	-.15	.32	.32	.99	.17	.34	.12	-.06	.50	-.24	.55	.59	.51
.05	.06	.06	-.04	-.19	-.09	.17	-.##	-.04	-.16	-.09	.45	-.09	.14	.18	-.17
.83	.84	.84	.47	.33	.32	.34	-.04	-.##	.34	.55	.36	.29	.89	.60	.60
.07	.07	.07	-.05	.25	.46	.12	-.16	.34	-.##	-.00	-.02	-.16	.18	.15	.14
.49	.49	.49	.72	.32	-.00	-.06	-.09	.55	-.00	-.##	.02	.45	.47	.24	.36
.48	.48	.48	-.00	-.01	.16	.50	.45	.36	-.02	.02	.99	-.03	.53	.56	.05
.21	.21	.21	.52	-.03	-.25	-.24	-.09	.29	-.16	.45	-.03	.99	.21	.00	.13
.92	.92	.92	.42	.32	.32	.55	.14	.89	.18	.47	.53	.21	-.##	.68	.63
.65	.68	.67	-.16	.24	.24	.59	.18	.60	.15	.24	.56	.00	.68	.99	.40
.76	.75	.75	.31	.49	.11	.51	-.17	.60	.14	.36	.05	.13	.63	.40	.99

ROW/COLUMN VARIBLES AS FOLLOWS

1 = TEMP AMB ABSOLUTE
2 = TEMP BRG ABSOLUTE
3 = TEMP WNG ABSOLUTE
4 = D*N (DIA. X RPM)
5 = NUMBER ON TEST
6 = NUMBER FAILED
7 = QUALITY CODE
8 = WEIBUL SLOPE
9 = TEST CORR COEF
10 = F-TEST VALUE
11 = WIRE SIZE
12 = NUMBER MOTOR POLES
13 = GREASE CODE
14 = TEST DATE
15 = SERIES
16 = LOG (CHARACTERISTIC LIFE)

CORR MATRIX WINDING FAILURES

Figure A-9

APPENDIX B

RECOMMENDED REPLACEMENT OF RELIABILITY MODEL FOR ELECTRONIC COOLING DEVICES AND FHP MOTORS FOR MIL-HDBK-217B

B.1 Motor Reliability Prediction Model

This section describes the method to be used for calculating failure rates of motors with power ratings below one horsepower. The types of motors that the predictive techniques were developed from include polyphase, capacitor start and run and shaded pole. Its application may be extended to other types of fractional horsepower motors utilizing rolling element grease packed bearings. The model is dictated by two failure modes, i.e.: bearing failures and winding failures. Application of the model to D.C. brush motors assumes that brushes are inspected and replaced and are not a failure mode. Motors included using these models cover applications of electronic cooling using fans and blowers as well as other motor applications. Reference (1*) contains a more comprehensive treatment of motor life prediction for motor applications where continuous operation at extremes of temperature, speed or load are anticipated. The reference should be reviewed when bearing loads exceed 10 percent of rated load, speeds exceed 24,000 rpm or motor loads include motor speed slip of greater than 25 percent.

Failure rates experienced by motors are not constant but increase with time. The failure rates described in the section are average rates obtained by dividing the cumulative hazard rate by the time period (t) of interest. Failure rate is most significantly influenced by operating temperature. Care should be exercised in using the best estimate of temperature for determination of failure rate.

---

*See Bibliography Number

The failure rate model is:

$$\lambda_t = \left[\frac{t_{sub}^{2}}{\alpha_B^{3}} + \frac{1}{\alpha_w}\right] x\ 10^6 \text{ (failures/}10^6\text{hours)} \qquad (1)$$

where: λ_t = the failure rate (failures/10^6 hours)

t_{sub} = time period for which average failure rate is calculated (hours)

t_1 = Estimated average failure rate at MTBF as tabulated in Table A calculated from Equation (4). Tables are used for constant temperature operation and equations (2), (3), and (4) used for cyclic temperature operation.

t_2 = If failure rate using t_1 is excessive relative to equipment requirements, user may select a smaller value of t; i.e.: t_2. All motors however, must be replaced at t_2 to make this failure rate valid. User selects this value, it is not calculated.

α_B = Bearing Weibull Characteristic Life as determined from Table A or Equation (2).

α_w = Winding Weibull Characteristic Life as determined from Table A or from Equation (3).

B.2 Predicting Failure Rate Without Utilizing Tables

Table A tabulates the Weibull characteristic bearing life and winding life (α_B and α_w, respectively). If tables are not used, the following procedure may be followed:

1. Determine operating ambient temperature in degrees centigrade. Calculate α_B and α_w from

$$\frac{1}{\alpha_B} = \frac{1}{10^{\left[\frac{2357}{T+273} - 2.534\right]}} + \frac{1}{10^{\left[\frac{-4500}{T+273} + 20\right]} + 300} \qquad (2)$$

MOTOR FAILURE RATE TABLE

TABLE A

AMBIENT TEMPERATURE (DEG C)	BEARING LIFE ALPHA B (HRS)	WINDING LIFE ALPHA W (HRS)	MOTOR FAILURE RATE NUMBER PER (MILLION HOURS)
140	1489	7533	574
138	1587	8031	538
136	1693	8567	504
134	1807	9144	472
132	1930	9766	442
130	2063	10438	414
128	2207	11163	387
126	2361	11947	361
124	2529	12794	338
122	2710	13711	315
120	2907	14704	294
118	3119	15780	274
116	3350	16948	255
114	3601	18215	237
112	3873	19591	220
110	4169	21088	205
108	4490	22717	190
106	4841	24490	176
104	5223	26423	163
102	5640	28532	151
100	6095	30834	140
98	6592	33350	129
96	7136	36102	119
94	7731	39114	110
92	8384	42416	101
90	9099	46037	93
88	9884	50013	86
86	10747	54382	79
84	11695	59188	73
82	12740	64481	67
80	13890	70315	61
78	15158	76753	56
76	16558	83865	51
74	18104	91729	47
72	19812	100434	43
70	21700	110082	39
68	23790	120787	35
66	26101	132679	32
64	28658	145903	29
62	31484	160627	27
60	34605	177042	24

(DEG C)	(HRS)	(HRS)	(MILLION HOURS)
58	38044	195364	22
56	41821	215840	20
54	45948	238754	18
52	50427	264428	16
50	55234	293233	15
48	60317	325596	14
46	65571	362005	12
44	70827	403024	11
42	75829	449303	11
40	80224	501592	10
38	83575	560761	10
36	85398	627815	9
34	85256	703921	9
32	82875	790439	9
30	78255	888951	10
28	71716	1001301	11
26	63838	1129648	12
24	55318	1276517	14
22	46812	1444875	16
20	38827	1638206	20
18	31677	1860614	24
16	25502	2116943	30
14	20316	2412923	38
12	16050	2755341	48
10	12598	3152259	62
8	9840	3613269	79
6	7657	4149814	102
4	5944	4775570	131
2	4609	5506924	169
0	3574	6363552	219
-2	2776	7369142	282
-4	2165	8552277	361
-6	1698	9947532	461
-8	1343	11596838	583
-10	1075	13551182	728
-12	873	15872719	897
-14	722	18637433	1084
-16	609	21938464	1286
-18	525	25890310	1491
-20	463	30634128	1689
-22	417	36344462	1875
-24	384	43237786	2036
-26	360	51583418	2174
-28	342	61717476	2285
-30	330	74060813	2370
-32	321	89142125	2439
-34	314	107627855	2489
-36	310	130360998	2525
-38	307	158411692	2549
-40	304	193143380	2570

$$\alpha_w = 10^{\frac{2357}{T+273} - 1.83} \qquad (3)$$

2. Utilizing the above equations for α_B and α_w, the mean time between failure (t) may be estimated from:

$$t = \alpha_B \left\{ \sqrt[3]{.34657 + \sqrt{(.34657)^2 + .03704\left[\frac{\alpha_B}{\alpha_w}\right]^3}} + \sqrt{.34657 - \sqrt{(.34657)^2 + .03704\left[\frac{\alpha_B}{\alpha_w}\right]^3}} \right\} \qquad (4)$$

3. Calculate the failure rate from Equation (1).

B.2.1 Bearing Characteristic Life Resulting from Thermal Cycling

$$\alpha_B = \frac{(h_1 + h_2 + h_3 + - - - - - - h_m)}{\frac{h_1}{\alpha_{B_1}} + \frac{h_2}{\alpha_{B_2}} + \frac{h_3}{\alpha_{B_3}} + - - - \frac{h_m}{\alpha_{B_m}}} \qquad (5)$$

where: (See Figure B-1)

h_1 = time at temperature T_1

h_2 = time to cycle from temperature T_1 to T_3

h_3 = time at temperature T_3

h_m = time at temperature T_m

α_{B_1} = bearing life at T_1 from Table A

α_{B_2} = bearing life at T_2 from Table A

α_{B_m} = bearing life at T_m from Table A

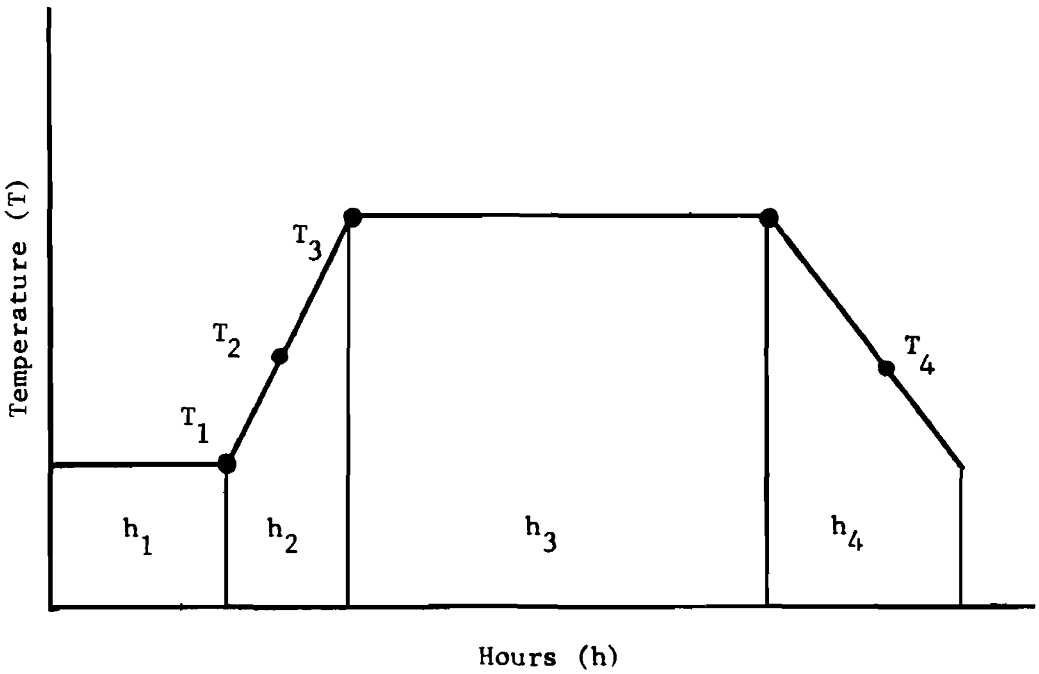


Figure B-1

Thermal Cycle

$$T_2 = \frac{T_1 + T_3}{2}$$

from example of Figure A-1,

$$T_m = T_4 \left[\frac{T_3 + T_1}{2}\right] \text{ and } h_m = h_4$$

B.3 Winding Characteristic Life Resulting from Thermal Cycling

$$\alpha_w = \frac{(h_1 + h_2 + h_3 + - - - - - - h_m)}{\frac{h_1}{\alpha_{w_1}} + \frac{h_2}{\alpha_{w_2}} + - - - - - - \frac{h_m}{\alpha_{w_m}}}$$

where: (See Figure A-1)

h_1 = time at temperature T_1

h_2 = time to cycle from temperature T_1 to T_3

h_3 = time at temperature T_3

h_m = time at temperature T_m

α_{w_1} = winding life at T_1 from Table A

α_{w_2} = winding life at T_1 to T_3 from Table A

α_{w_m} = winding life at T_m from Table A

$T_2 = \frac{T_1 + T_3}{2}$

from example of Figure A-1 $T_m = T_4 = \frac{T_3 + T_1}{2}$ and $h_m = h_4$

B.4 Examples of Motor Failure Rate Prediction

B.4.1 Example No. 1. Given: Fractional Horsepower Motor operating at an ambient temperature of 120°C. Find failure rate for replacement of motors in 1000 hours. Also find failure rate at estimated mean time between failures.

a) For replacement of motors in 1000 hours:
From Table A at 120°C

$$\alpha_B = 2907 \text{ hours}$$

$$\alpha_W = 14704 \text{ hours}$$

From Equation (1)

$$\lambda_t = \left[\frac{t^2}{\alpha_B^{\;3}} + \frac{1}{\alpha_W}\right] \times 10^6$$

$$\lambda_t = \left[\frac{1000^2}{2907^3} + \frac{1}{14704}\right] \times 10^6 = 108.7 \text{ failures}/10^6 \text{ hours}$$

b) For failure rate at estimated MTBF
From Table 1 at 120°C

$$\lambda_t = 294 \text{ failures}/10^6 \text{ hours}$$

B.4.2 Example No. 2 Given: Fractional Horsepower Motor operating at a thermal duty cycle of:

2 hours at 100°C ambient
8 hours at 20°C ambient
0.5 hours to change temperature

Find average failure rate at MTBF

a) Tabulate operating temperatures:

T_1 = 100°C; h_1 = 2 hours

$$T_2 = \frac{100 + 20}{2} = 60^{o}C; \quad h_2 = 1 \text{ hour}$$

$$T_3 = 20^{o}C; \quad h_3 = 8 \text{ hours}$$

b) Determine bearing and winding life from Table A

$$T_1 = 100^{o}C; \quad \alpha_B = 6095 \text{ hours}; \quad \alpha_w = 30834 \text{ hours}$$

$$T_2 = 60^{o}C; \quad \alpha_B = 34605 \text{ hours}; \quad \alpha_w = 177042 \text{ hours}$$

$$T_3 = 20^{o}C; \quad \alpha_B = 38827 \text{ hours}; \quad \alpha_w = 1638206 \text{ hours}$$

c) Calculate bearing life and winding life from Equations (5) and (6)

$$\alpha_B = \frac{2 + 1 + 8}{\frac{2}{6095} + \frac{1}{34605} + \frac{8}{38827}} = 19535 \text{ hours}$$

$$\alpha_w = \frac{2 + 1 + 8}{\frac{2}{30834} + \frac{1}{177042} + \frac{8}{1638206}} = 145899 \text{ hours}$$

d) Calculate (t) from Equation (4)

$$t = 19535 \left[\sqrt[3]{.34657 + \sqrt{.1201 + .0370 \left(\frac{19535}{145899}\right)^3}} + \sqrt[3]{.34657 - \sqrt{.12 + .03704 \left(\frac{19535}{145899}\right)^3}} \right]$$

$$= 19535 \quad \sqrt[3]{.69325} + \sqrt[3]{-.00011}$$

$$= 19535 \left[.88504 - .0483\right]$$

$$t = 16346 \text{ hours}$$

e) Calculate average failure rate at MTBF from Equation (1)

$$\lambda_t = \left[\frac{t^2}{\alpha_B{}^3} + \frac{1}{\alpha_w}\right] \times 10^6$$

$$\lambda_t = \left[\frac{(16346)^2}{(19535)^3} + \frac{1}{195899}\right] \times 10^6$$

$$\lambda_t = 42.7 \text{ failures}/10^6 \text{ hrs}$$

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